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# SUBSONIC WIND TUNNEL TESTING HANDBOOK



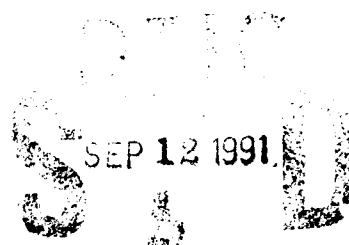
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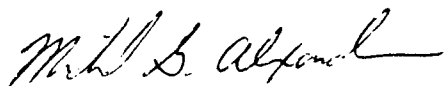
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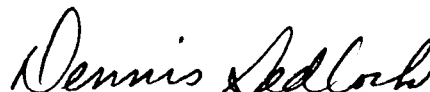
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## Foreword

This technical report was prepared by Captain Michael G. Alexander from Wright Laboratory, Flight Dynamics Directorate, Aeromechanics Division, Wright-Patterson AFB, Ohio, 45433-6553. This work was accomplished under work project 240410A2, Advanced Tactical Transport. This technical report provides the wind tunnel testing engineer a handbook with useful equations, definitions, and concepts to aid in wind tunnel testing.

The author wishes to thank the exceptional insights, observations, expertise and help from Mr. Tom Tighe, Mr. Jim Grove, Mr. Bob Guyton and Mr. Larry Rogers all from WL/FIMMC (Wright Laboratory, Airframe Aerodynamics Group).

This report has been reviewed and is approved.

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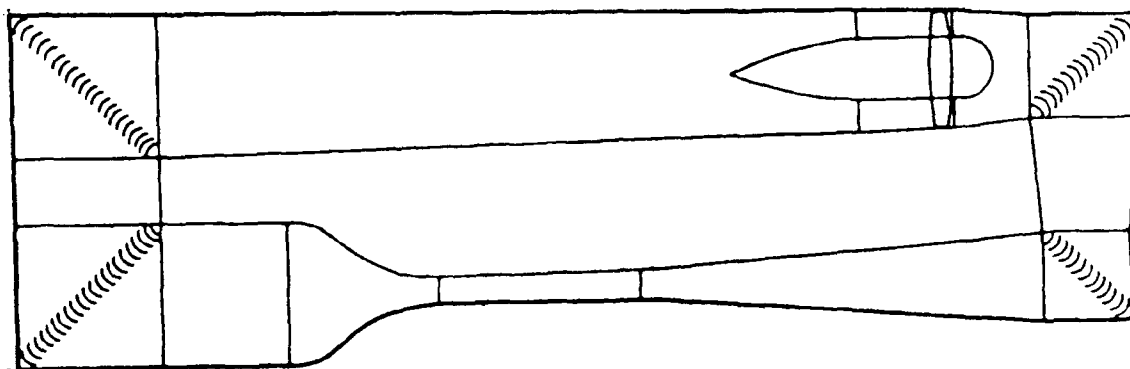
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**W**ind

**T**unnel

**T**esting

**H**andbook



By Captain Michael G. Alexander, USAF



# I INTRODUCTION

## I Introduction

The aerodynamic engineer who participates in wind tunnel testing often has a need for reference material to aid him/her in his/her testing. It is nearly impossible for the engineer to recall every equation, concept, definition, and to carry reference material to the wind tunnel site. This handbook, Subsonic Wind Tunnel Testing Handbook, attempts to provide to the testing engineer in one reference some of those equations, concepts, and definitions that he or she might find helpful during testing. This handbook is a quick reference for the testing engineer and is an amalgamation of information from numerous reference materials and personal observations. It is designed to be a living document readily expandable and to fit in a briefcase to accompany the testing engineer to the test site. Also by design, this aid has not encompassed every wind tunnel technique or principle, but offers enough information to facilitate and help ease wind tunnel testing. This handbook is predominantly structured for subsonic (non-compressible flow), force and moment, wind tunnel testing. For compressible flow testing, an excellent reference aid is the NACA 1135 (ref. 1). However, it is no longer in print by the government, but, reprints of the NACA 1135 can be procured from reference 2.

## II AERODYNAMIC DEFINITIONS

## II Aerodynamic Definitions

### Symbols

a.c.	Aerodynamic Center
cg	Center of Gravity
$C_L$	Lift Coefficient
$C_n$	Yawing Moment Coefficient
$C_M$	Pitching Moment Coefficient
$C_{M_o}$	Pitching Moment Coefficient at zero angle-of-attack
cp	Center of Pressure
MAC	Mean Aerodynamic Chord
$N_o$	Neutral Point
SM	Static Margin
$X_{np}$	Distance from the MAC leading edge to the aerodynamic center
$X_{cg}$	Distance from the MAC leading edge to the center of gravity
$\frac{\partial C_m}{\partial \alpha}$	Pitching Moment Coefficient slope
$\frac{\partial C_{mcg}}{\partial C_L}$	Slope of the cg pitching moment and lift curve
$\frac{\partial C_n}{\partial \phi}$	Slope of yawing moment coefficient and the yaw angle
$\frac{\partial C_n}{\partial \beta}$	Slope of yawing moment coefficient and the sideslip angle
$\frac{\partial C_l}{\partial \beta}$	Slope of rolling moment coefficient and the sideslip angle
$\alpha$	Angle-of-Attack
$\beta$	Sideslip Angle
$\phi$	Yaw Angle

Aerodynamic Center (a.c.) - The point where the pitching moment does not vary with the angle of attack.

Center of Pressure (cp)- A point where the pitching moment vanishes. The cp location varies with angle of attack and Mach number.

Mean Aerodynamic Chord (MAC) - The chord of an imaginary, untwisted, unswept, equal span non-tapered wing which would have force vectors throughout the flight range identical with those of the actual wing or wings.

Neutral Point ( $x_{np}$  or  $N_o$ ) - As the cg is moved aft, the slope of  $\partial C_{m_{cg}} / \partial C_L$  becomes less negative. When there is no change of the pitching moment with lift coefficient ( $\partial C_{m_{cg}} / \partial C_L = 0$ ), that  $x_{cg}$  position is the neutral point ( $N_o = x_{cg}$ ).

Static Margin - The distance from the center of gravity to the neutral point expressed as a fraction of the mean aerodynamic chord. The 'X' position of  $x_{np}$  is measured positive aft from the c.g..

$$SM = \frac{X_{np} - X_{cg}}{\bar{c}} = \frac{\partial C_{m_{cg}}}{\partial C_L}$$

$$+SM = \frac{\partial C_{m_{cg}}}{\partial C_L} < 0 \quad (\text{Stable; } N_o \text{ is behind c.g.})$$

$$-SM = \frac{\partial C_{m_{cg}}}{\partial C_L} > 0 \quad (\text{Unstable; } N_o \text{ is ahead of c.g.})$$

Longitudinal Static Stability - Is determined by the sign and the magnitude of the slope of  $C_m$  versus  $\alpha$  curve.  $\frac{\partial C_m}{\partial \alpha}$  must be  $< 0$  and  $C_{m_o}$  must be positive for trimmed, positive longitudinal static stability. Also,  $C_m$  versus  $C_L$  can be used to determine

positive longitudinal stability.

Directional Static Stability - Is determined by the sign and magnitude of the slope of  $C_n$  versus  $\beta$  curve, where  $\beta$  is the sideslip angle.  $\frac{\partial C_n}{\partial \beta}$  must be positive ( $> 0$ ) for directional stability. This is also known as weathercock stability. Also,  $C_n$  versus  $\phi$  can be used where  $\phi$  is the yaw angle.  $\frac{\partial C_n}{\partial \phi}$  must be negative ( $< 0$ ) for directional stability.

Lateral Static Stability - It is determined by the sign and magnitude of the slope of the  $C_l$  versus  $\beta$  curve.  $\frac{\partial C_l}{\partial \beta}$  must be negative ( $< 0$ ) for positive lateral stability. This is also known as the dihedral effect. An increase in lateral stability causes dutch roll; too little stability causes spiral instability.

Dynamic Stability - Is the time history of the movements of a body in response to its static stability tendencies following an initial disturbance from equilibrium.

Trimmed Flight - For balanced flight, the aircraft flies at a given elevator angle and angle of attack that produces  $C_m = 0$ .

Critical Mach Number - Is the Mach number where there is a sharp increase in drag (drag divergence, approximately Mach = 1.0).

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## NOTES

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### III    FORCE AND MOMENT EQUATIONS

### III Force and Moment Equations

#### General Aerodynamic Symbols and Equations

A	Axial Force (lbf) -- $(A = D\cos\alpha - L\sin\alpha)$
a	Speed of Sound (f/s) -- $\sqrt{(\gamma R \mathcal{T})} = 49\sqrt{\mathcal{T}} = \sqrt{(\gamma P/\rho)}$ ; $\mathcal{T} = {}^\circ\text{R}$ Speed of Sound (mph) -- $33.42\sqrt{\mathcal{T}}$ ; $\mathcal{T} = {}^\circ\text{R}$
a	Lift Curve Slope -- $C_{L\alpha}$
$C_A$	Axial Force Coefficient ( $C_A = -C_L\sin\alpha + C_D\cos\alpha$ )
$C_D$	Drag Coefficient ( $C_D = C_N\sin\alpha + C_A\cos\alpha$ )
$C_{D_i}$	Induced Drag
$C_{D_o}$	Zero Lift Drag; Parasite Drag (profile, friction, pressure)
$C_L$	Lift Coefficient ( $C_L = C_N\cos\alpha - C_A\sin\alpha$ )
$C_L'$	Lift at minimum drag (generally zero)
$C_l$	Rolling Moment
$C_{L\alpha}$	Lift Curve Slope -- $\frac{\delta C_L}{\delta \alpha}$
$C_m$	Pitching Moment
$C_{m\alpha}$	Pitching moment curve slope -- $\frac{\delta C_m}{\delta \alpha}$
$C_N$	Normal Force Coefficient ( $C_N = C_L\cos\alpha + C_D\sin\alpha$ )
$C_n$	Yawing Moment
$C_{n\beta}$	Yawing moment curve slope -- $\frac{\delta C_n}{\delta \beta}$
$C_T$	Thrust Coefficient -- $\frac{\text{Thrust}}{q S_w}$
cp	Center of Pressure -- $\frac{x_{cp}}{\bar{c}} = -\frac{C_m}{C_N}$

# General Aerodynamic Symbols and Equations (continued)

$C_Y$	Side Force Coefficient
D	Drag Force (lbf) -- $(D = N\sin\alpha + A\cos\alpha)$
	Drag Polar -- $C_D = C_{D_o} + \frac{[C_L - C_L']^2}{\pi eAR}$
e	Oswald's wing efficiency factor
g	Gravity Constant
K	Induced drag factor -- $\frac{1}{\pi eAR}$
L	Lift Force (lbf) -- $(L = N\cos\alpha - A\sin\alpha)$
N	Normal Force -- $(L\cos\alpha + D\sin\alpha)$ (lbf)
$N_o$	Neutral point -- $\frac{\delta C_m}{\delta C_L} = \frac{x_{cg} - N_o}{\bar{c}}$
n	Load Factor -- $\frac{L + T\sin\alpha_T}{W} = \frac{L}{W}$ (for $\alpha_T$ small)
M	Mach number
PM	Pitching Moment (dimensional)
P	Pressure
R	Gas Constant
T	Thrust
$\mathcal{T}$	Temperature; ( $^{\circ}$ Rankine)
q	Dynamic Pressure -- $0.5\rho V^2$ or $0.7(P_s)M^2$
RM	Rolling Moment (dimensional)
S	Surface Area
S	Side Force (lbf)
$\bar{V}$	Tail/Canard Volume Coefficient = $\left[ \frac{l_t S_t}{q_{\infty} S_w \bar{c}} \right]$

### General Aerodynamic Symbols and Equations (concluded)

$X_{np}$	Neutral point -- $\frac{\delta C_m}{\delta C_L} = \frac{x_{cg} - N_o}{\bar{c}}$
Y	Yawing Force (side force)
YM	Yawing Moment (dimensional)
$\alpha_T$	Thrust vector angle
$\alpha$	Geometric angle of attack (measured from $V_\infty$ to the wing chord)
$\alpha_a$	Absolute angle of attack ( $\alpha - \alpha_{ZL}$ )
$\alpha_{ZL} ; \alpha_o$	Zero lift angle of attack (measured from the wing chord to the wing zero lift line)
$\beta$	Angle of sideslip
$\phi$	Angle of roll
$\eta$	Tail efficiency factor ( $q_t/q_\infty$ )
$\varphi$	Angle of yaw
$\epsilon$	Downwash angle
$\theta$	local angle of inclination of surface to $V_\infty$

### Subscripts

np -- neutral point	w -- wing	t -- tail
s -- static pressure	T -- Thrust	c -- canard
cg -- center of gravity	ac -- aerodynamic center	
inc -- incompressible	$\infty$ -- freestream conditions	

Table 1  
Force and Moment Definitions

Forces

Body Axis		Wind Axis		Stability Axis	
Force	Coefficient	Force	Coefficient	Force	Coefficient
N	$C_N = N/qS_w$	L	$C_L = L/qS_w$	L	$C_L = L/qS_w$
A	$C_A = A/qS_w$	D	$C_D = D/qS_w$	D	$C_D = D/qS_w$
S	$C_S = S/qS_w$	Y	$C_Y = Y/qS_w$	Y	$C_Y = Y/qS_w$

Moments

Body Axis		Wind Axis		Stability Axis	
Moment	Coefficient	Moment	Coefficient	Moment	Coefficient
PM	$C_m = PM/qS_w \bar{c}$	PM	$C_m = PM/qS_w \bar{c}$	PM	$C_m = PM/qS_w \bar{c}$
RM	$C_l = RM/qS_w b$	RM	$C_l = RM/qS_w b$	RM	$C_l = RM/qS_w b$
YM	$C_n = YM/qS_w b$	YM	$C_n = YM/qS_w b$	YM	$C_n = YM/qS_w b$

Parameter legend

$q$  = Dynamic pressure     $b$  = Span     $S_w$  = Wing area     $\bar{c}$  = MAC



## Aerodynamic Equations

### Trim Pitching Moment Equations (ref. 3)

#### Conventional Horizontal Tailed Aircraft (figure III-1)

$$C_{M_{cg}} = [C_{M_{ac}}]_w + C_L \left[ \frac{X_w}{\bar{c}} \right] + C_D \left[ \frac{z}{\bar{c}} \right] + \left[ \frac{T}{q_\infty S_w \bar{c}} Z_T \right] - C_{L_t} (\bar{V}) \eta_t - [C_{M_{cg}}]_{inlet}$$

For an all flying tail....

$$C_{L_t} = a_t (\alpha_t + \Delta\alpha)$$

$$= a_t \left[ \left( 1 - \frac{\delta\epsilon}{\delta\alpha} \right) \alpha + \Delta\alpha \right]$$

$\Delta\alpha = \alpha$  created by control column input

For fixed stabilizer and a movable elevator...

$$C_{L_t} = a_t \left[ \left( 1 - \frac{\delta\epsilon}{\delta\alpha} \right) \alpha - (\alpha_{ZL})_t \right]$$

#### Tailless Aircraft (figure III-2)

$$C_{M_{cg}} = [C_{M_{ac}}]_w - C_L \left[ \frac{X_w}{\bar{c}} \right] + C_D \left[ \frac{z}{\bar{c}} \right] + \left[ \frac{T}{q_\infty S_w \bar{c}} Z_T \right] - [C_{M_{cg}}]_{inlet}$$

#### Canard Aircraft (figure III-3)

$$C_{M_{cg}} = [C_{M_{ac}}]_w - C_L \left[ \frac{X_w}{\bar{c}} \right] + C_D \left[ \frac{z}{\bar{c}} \right] + \left[ \frac{T}{q_\infty S_w \bar{c}} Z_T \right] - C_{L_c} [\bar{V}_c] - [C_{M_{cg}}]_{inlet}$$

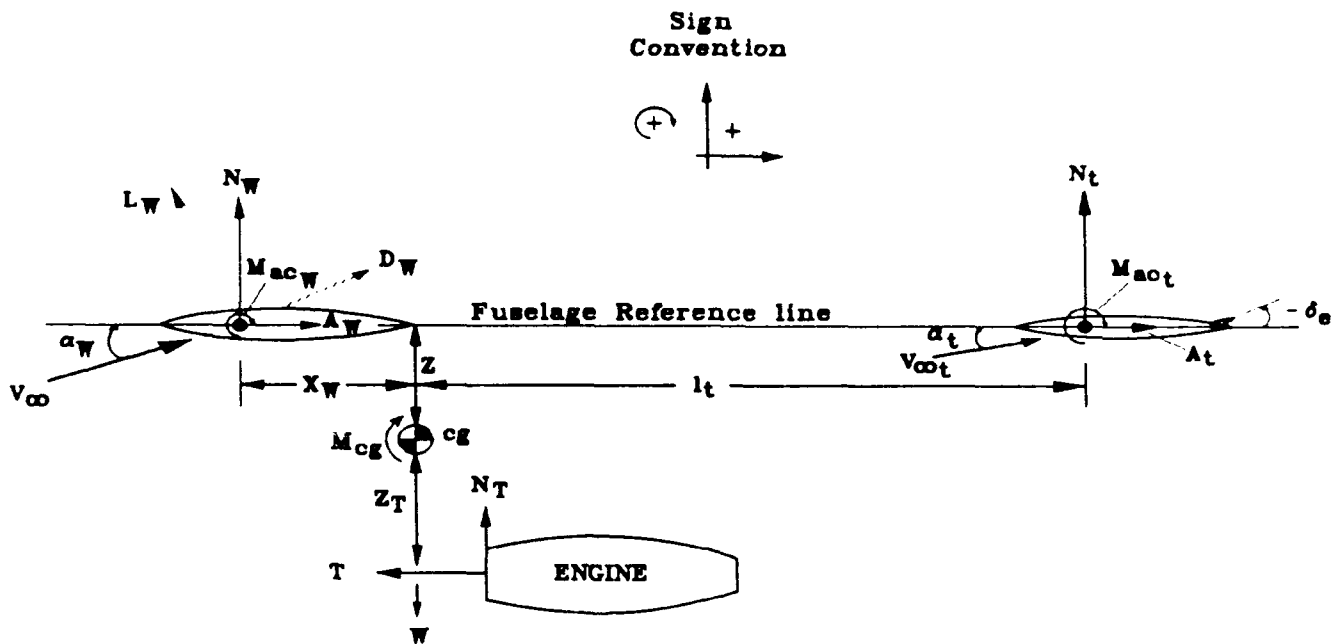


Figure III-1 Force and Moment Vectors  
Aft Tailed Aircraft

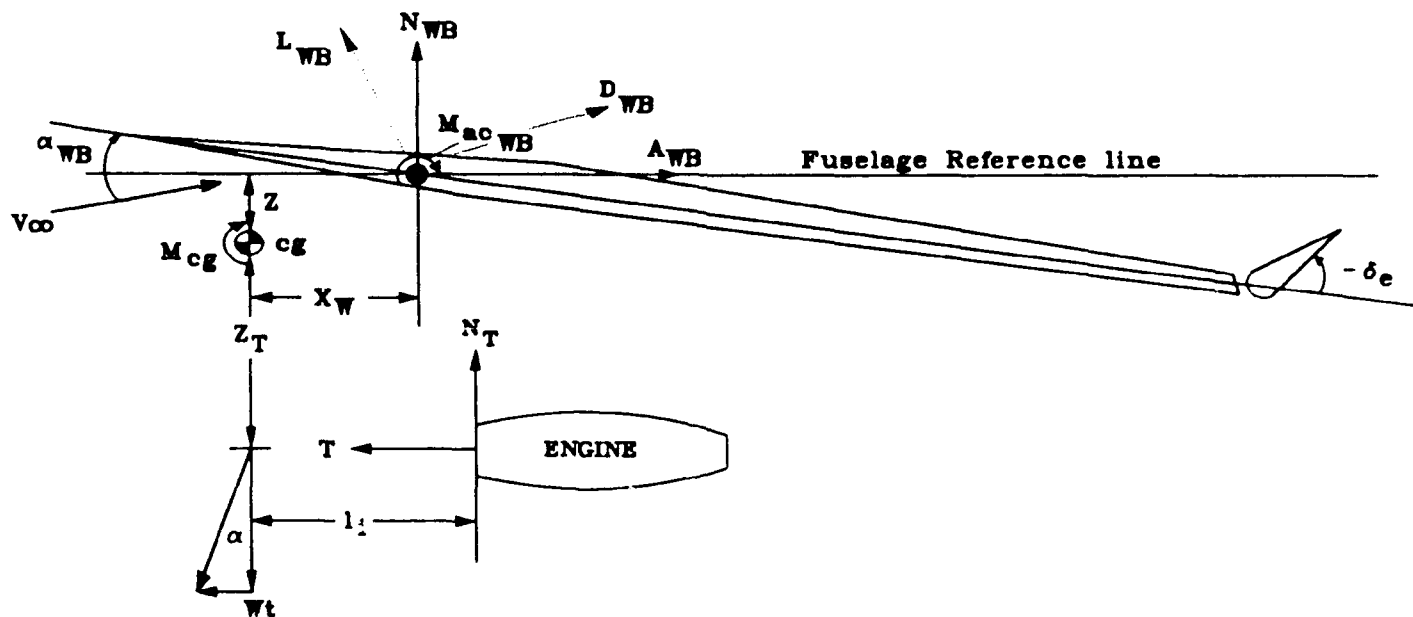
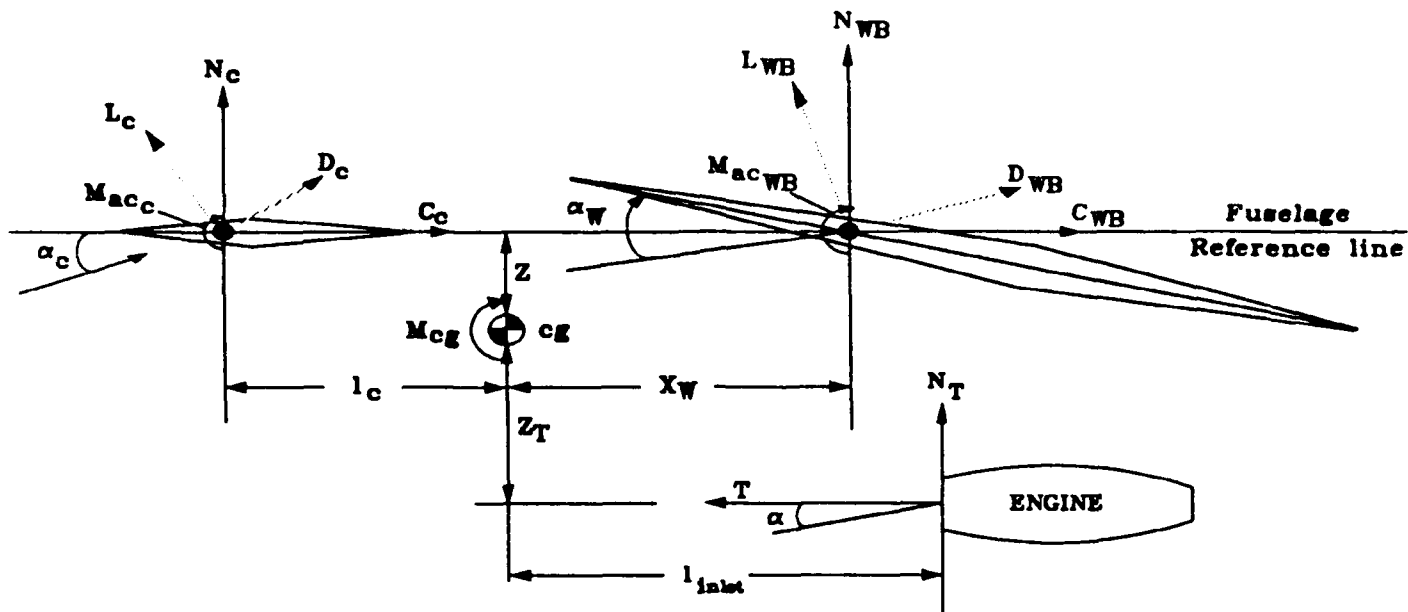


Figure III-2 Force & Moment Vectors  
Tailless Aircraft



### Canard Aircraft

Figure III-3 Force and Moment Vectors  
Canard Aircraft

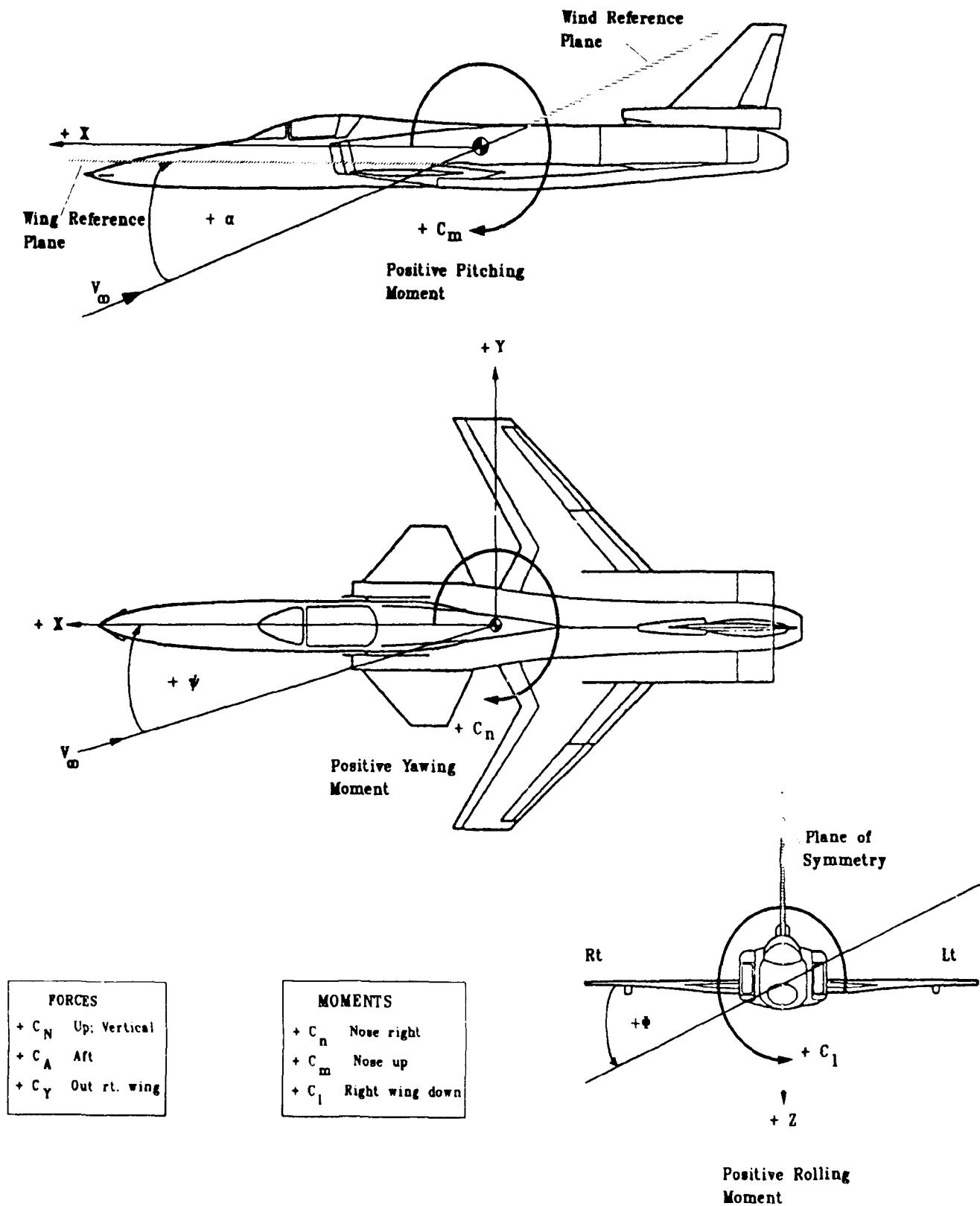


Figure III-4 Positive Angles, Moments, and Body Axis System

### Stick-Fixed Neutral Point

$$N_o = X_{cg} \quad ; \text{when } (\delta C_M / \delta C_L) = 0$$

$$N_o = X_{ac} - \left[ \delta C_M / \delta C_L \right] + (a_t / a_w) \bar{V} \eta_t [1 - (\delta \epsilon / \delta \alpha)]$$

$$\delta C_M / \delta C_L = \frac{x_{cg} - N_o}{\bar{c}}$$

### Two-Dimensional Lift

#### Subsonic

$$C_{L_{2D_{inc}}} = \frac{a_{inc} (\alpha_a)}{\sqrt{1 - M^2}} \quad (\alpha \text{ in rads})$$

#### Supersonic

$$C_{L_{2D}} = \frac{C_{L_{2D_{inc}}}}{\sqrt{M^2 - 1}} \quad C_{L_{2D}} = \frac{4\alpha}{\sqrt{M^2 - 1}} \quad \begin{array}{l} \text{(flat plate)} \\ (\alpha \text{ in rads}) \end{array}$$

### Pressure Coefficient

#### Incompressible

$$C_{P_{inc}} = 1 - \left[ \frac{V}{V_\infty} \right]^2 \quad C_{P_{inc}} = \frac{P - P_\infty}{q_\infty} \quad C_P = \frac{C_{P_{inc}}}{\sqrt{1 - M^2}} \quad \begin{array}{l} \text{(thin-airfoil} \\ \text{theory)} \end{array}$$

#### Compressible

$$C_P = \frac{2}{\gamma M_\infty^2} \left[ \left[ \frac{1 + .5(\gamma - 1)M_\infty^2}{1 + .5(\gamma - 1)M^2} \right]^{\frac{\gamma}{\gamma - 1}} - 1 \right]$$

$$C_p = \frac{2\theta}{\sqrt{M^2 - 1}} \quad \begin{array}{ll} \theta = >0 & \text{compression} \\ \theta = <0 & \text{expansion} \end{array} \quad (\theta \text{ in rads})$$

### Center of Pressure Location

$$X_{cp} = - \frac{C_M}{C_N} \text{ (units of length)}$$

### Aerodynamic Center (a.c.) Determination

Assume the moment reference center is at a distance 'x' from the L.E.  
and then take moments about the a.c. ....

$$C_{M_{ac}} q \bar{c} S = \left[ C_{M_x} q \bar{c} S \right] + \left[ C_L q S \right] (x_{ac} - x) \cos \alpha + \left[ C_D q \bar{c} S \right] (x_{ac} - x) \sin \alpha$$

Solving for  $x_{ac}$  . . .

$$\frac{x_{ac}}{\bar{c}} = \frac{x}{\bar{c}} - \frac{C_{M_x} - C_{M_{ac}}}{C_L \cos \alpha + C_D \sin \alpha}$$

for  $\alpha \ll 1$

$$\frac{x_{ac}}{\bar{c}} = \frac{x}{\bar{c}} - \frac{C_{M_x} - C_{M_{ac}}}{C_L}$$

### Mass Flow (ref. 27)

$$M_o = \rho A V$$

## NOTES

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## IV    AXIS SYSTEM DEFINITIONS

#### IV Axis System Definitions

(Ref. 6)

Tunnel Axis - An orthogonal, right-handed axis system which remains fixed with respect to the tunnel in pitch, roll and yaw.

Body Axis - An orthogonal, right-handed axis system which remains fixed with respect to the model and rotates with it in pitch, roll, and yaw (figure IV-1). All forces, moments, and axis systems are referenced from the Body axis.

$x_b$  - Longitudinal body axis in the plane of symmetry of the aircraft; positive forward.

$y_b$  - Lateral body axis perpendicular to the plane of symmetry of the aircraft, usually taken in the plane of the wing; positive toward right wing tip.

$z_b$  - Vertical body axis in the plane of symmetry of aircraft, perpendicular to the longitudinal and lateral axes; positive down.

Wind Axis - An orthogonal, right-handed axis system which is obtained by rotating through pitch and yaw, but not roll with respect to the Body axis. This system defines lift as perpendicular to the relative wind, drag as parallel to the relative wind, and side force as perpendicular to the plane of lift and drag (figure IV-2).

Stability Axis - An orthogonal, right-handed axis system which remains fixed with respect to the relative wind in pitch, but rotates with the model in yaw and roll. Lift is defined as perpendicular to the relative wind with drag and side force yawing with the model (figure IV-3). The only difference between the Stability axis and the Body axis is  $\alpha$ .

$x_s$  - Longitudinal stability axis, parallel to the projection of the total velocity vector ( $\bar{V}$ ) on the plane of symmetry of the aircraft; positive forward

$y_s$  - Lateral stability axis, coincident as the lateral body axis; positive out the right wing tip.

$z_s$  - Vertical stability axis in the plane of symmetry of the aircraft, perpendicular to the longitudinal and lateral stability axes; positive down.

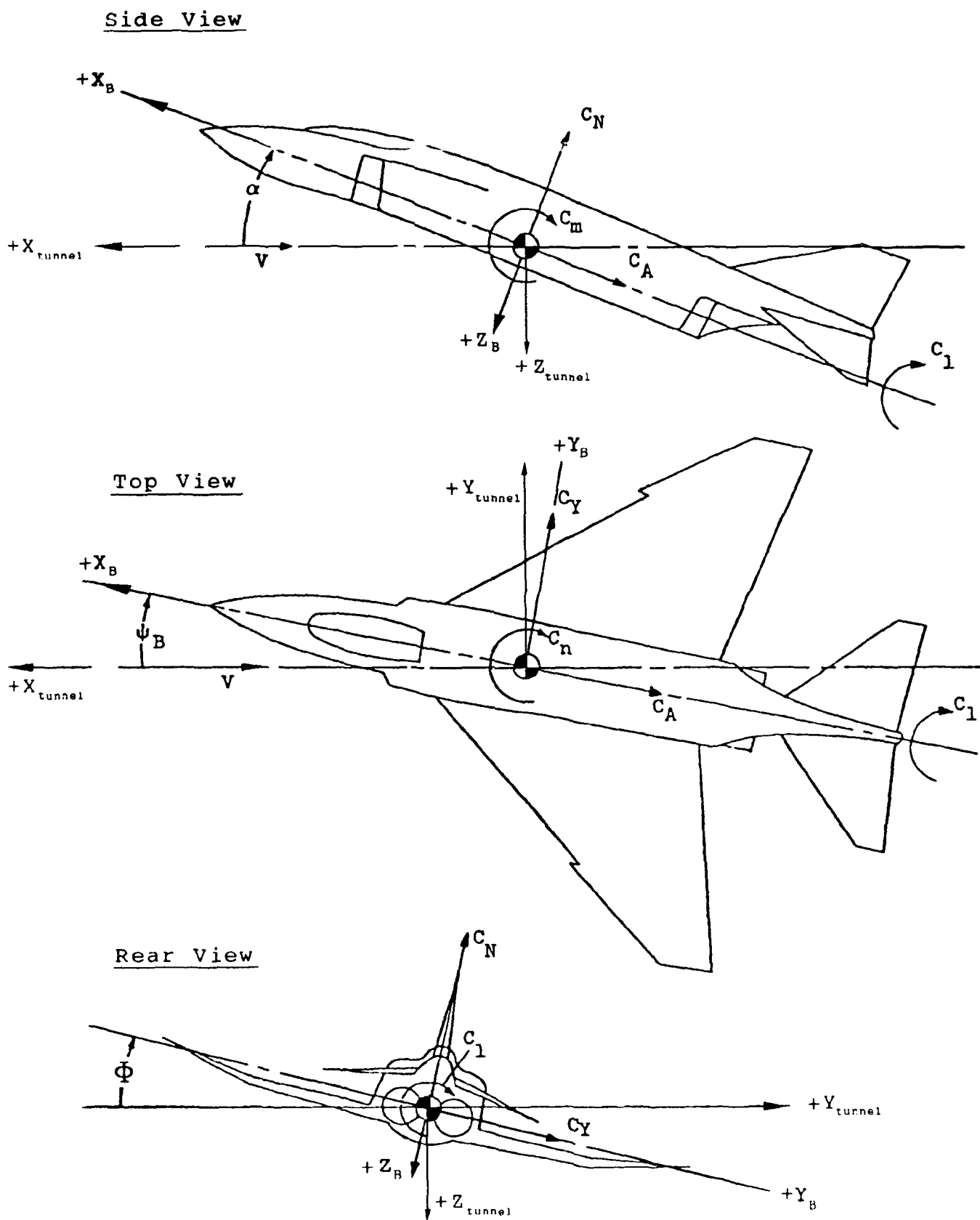


Figure IV-1 Body Axis System

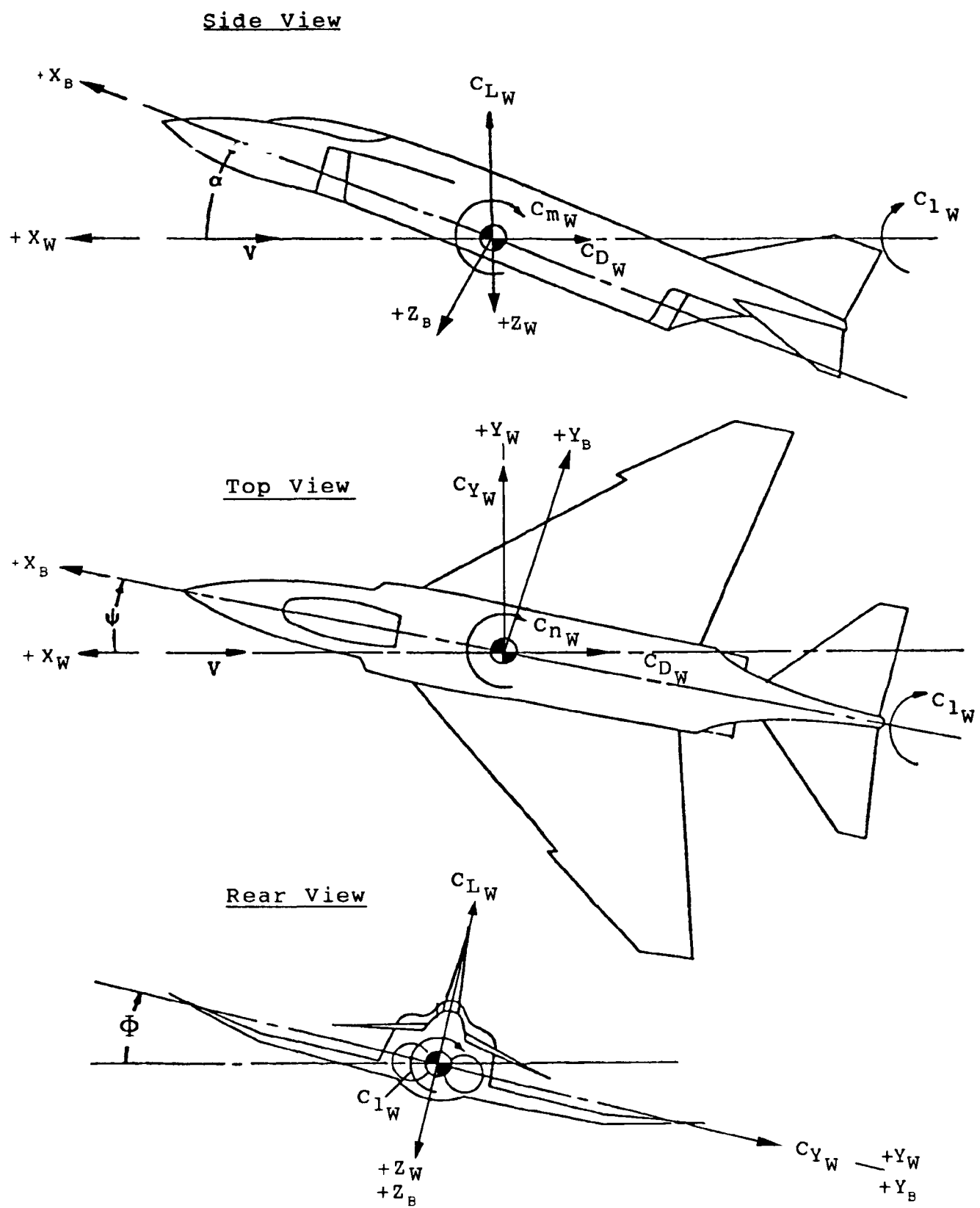


Figure IV-2 Wind Axis System



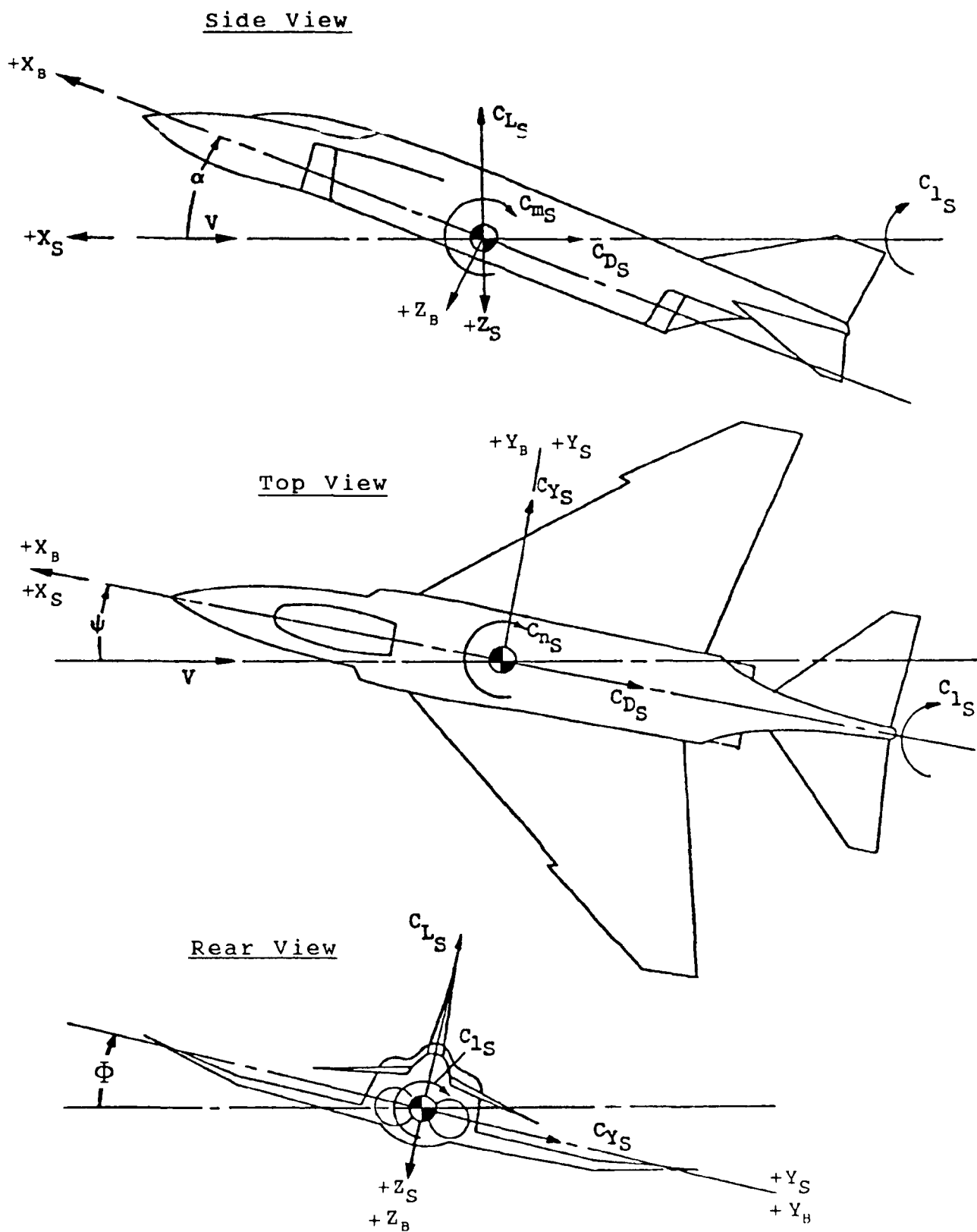


Figure IV-3 Stability Axis System

## Angle Definitions (figure IV-4)

### Aerodynamic Angles

$\alpha$  - Pitch angle-of-attack, angle between the  $x_b$  axis and the projection of  $(\bar{V})$  on the plane of symmetry. A positive  $\alpha$  is pitch up or rotates the  $^+z_b$  axis into  $^+x_b$  axis.  $\alpha = \tan^{-1}(w/u)$

$\varphi$  - Yaw angle, angle between the projected total velocity vector  $(\bar{V})$  on the  $x_b y_b$  plane and the  $^+x_b$  axis. A positive direction rotates the  $^+x_b$  axis into the  $^+y_b$  axis (positive nose right).  $\varphi = \tan^{-1}(v/u)$ .

$\beta$  - Angle-of-sideslip, angle between the total velocity vector  $(\bar{V})$  and its projection on the  $x_b z_b$  plane. A positive rotation rotates the  $^+y_b$  axis into  $^+x_b$  axis (positive nose left).  $\beta = \sin^{-1}(v/\bar{V})$ .

### Orientation Angles (looking from origin; figure IV-4)

$\theta$  - Pitch angle, positive clockwise about the  $^+y_i$  axis direction (+ nose up;  $^+z_i$  into  $^+x_i$ ).

$\varphi$  - Yaw angle, positive clockwise about the  $^+z_i$  axis direction (+ nose right;  $^+x_i$  into  $^+y_i$ ).

$\phi$  - Roll angle, positive clockwise about the  $^+x_i$  axis direction (+ rt wing down;  $^+y_i$  into  $^+z_i$ ).

$\varphi$ ,  $\theta$ , and  $\phi$  form a system of three angles which defines the orientations of the Body axis with the respect to the Tunnel axis system (inertial reference system). Any orientation of the Body axis system is obtained by rotationally displacing it from the Tunnel axis system through each of the three angles in turn. The order of rotation is important (figure IV-5) and is defined to be yaw-pitch-roll.

A long discussion ensued over the definition of yaw ( $\varphi$ ) and sideslip ( $\beta$ ). After many

arguments and three dimensional velocity box drawings,  $\phi = -\beta$  (after yawing and pitching the model. Roll = 0). When the model is yawed, pitched, and rolled, then yaw does not equal minus sideslip ( $\phi \neq -\beta$ ). The difference is obviously roll.

#### Angle Transformation from Tunnel Axis (Inertial Axis) to Body Axis

(figure IV-4; +Z down, +X out nose, +Y out rt wing)

$$\begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ \sin\phi\sin\theta\cos\phi & \sin\phi\sin\theta\sin\phi & \sin\phi\cos\theta \\ -\cos\phi\sin\theta & +\cos\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}$$

b = body                      i = inertial

#### Wing Reference Plane (ref. 4)

It is the plane which passes through the wing tips and is parallel to the longitudinal axis.

#### Wind Reference Plane (ref. 4)

It is the plane that passes through the relative wind vector and intersects the wing reference plane along a line which is perpendicular to the plane of symmetry.

#### Plane of Symmetry (ref. 4)

It is the plane that passes through the fuselage axis and is perpendicular to both the wing and wind reference planes.

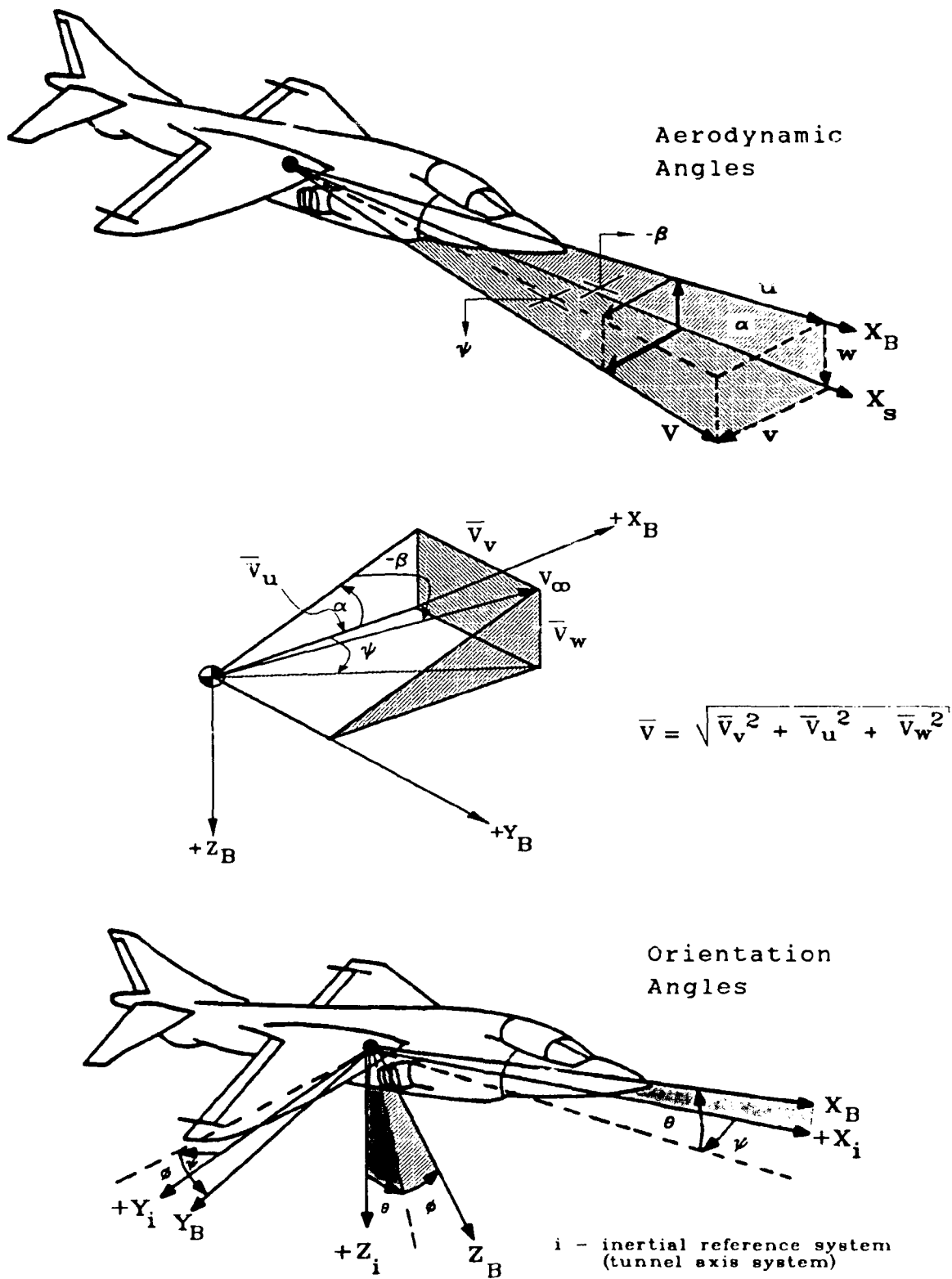


Figure IV-4 Angle Definitions

### Coordinate Transformation Equations

Most external balances measure about the wind axis system and most internal balances measure about the body axis system. Thus it becomes necessary to transfer from one axis system to another. If the model and balance are fixed to the sting, with no relative motion between the model and the balance, and the sting is capable of movement in yaw, pitch, and roll then body axis coincides with the balance axis. Therefore, all forces and moments indicated by the balance are body forces and moments. The axis can pass through either the balance moment reference center or the desired model c.g.. However, if the model and balance axes do not coincide, additional transformations must be considered.

The coordinate transformations below assume data have been transferred to the desired c.g. location. Also,  $\alpha$  is  $\alpha_{\text{body}}$  and has not been corrected for the wind tunnel upflows and wall effects.

### Balance Axis to Body Axis

(Ref. 28)

Order of rotation: yaw, pitch, roll (See figure IV-6)

Note:  $\alpha$  and  $\beta$  are balance to body axis angles and not aerodynamic angles.

Forces (roll = 0)

$$C_{N_b} = C_{N_{bal}} \cos \alpha + C_{A_{bal}} \cos \beta \sin \alpha - C_{Y_{bal}} \sin \beta \sin \alpha$$

$$C_{A_b} = C_{A_{bal}} \cos \beta \cos \alpha - C_{Y_{bal}} \sin \beta \cos \alpha - C_{N_{bal}} \sin \alpha$$

$$C_{Y_b} = C_{Y_{bal}} \cos \beta + C_{A_{bal}} \sin \beta$$

Forces (roll = 0)

$$\begin{bmatrix} C_{N_b} \\ C_{A_b} \\ C_{Y_b} \end{bmatrix} = \begin{bmatrix} \cos \alpha & \cos \beta \sin \alpha & \sin \beta \sin \alpha \\ -\sin \alpha & \cos \alpha \cos \beta & \cos \alpha \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} C_{N_{bal}} \\ C_{A_{bal}} \\ C_{Y_{bal}} \end{bmatrix}$$

Forces (roll  $\neq 0$ )

$$\begin{bmatrix} C_{N_b} \\ C_{A_b} \\ C_{Y_b} \end{bmatrix} = \begin{bmatrix} \cos \alpha \cos \phi & \sin \alpha \cos \beta \cos \phi & \sin \alpha \sin \beta \cos \phi \\ -\sin \alpha & \cos \alpha \cos \beta & \cos \alpha \sin \beta \\ -\cos \alpha \sin \phi & -\sin \alpha \cos \beta \sin \phi & -\sin \alpha \sin \beta \sin \phi \end{bmatrix} \begin{bmatrix} C_{N_{bal}} \\ C_{A_{bal}} \\ C_{Y_{bal}} \end{bmatrix}$$

Moments (roll  $\neq 0$ )

(Note: Body c.g. is located +X, +Y, +Z from balance c.g. See figure IV-6)

$$\begin{bmatrix} C_{l_b} \\ C_{m_b} \\ C_{n_b} \end{bmatrix} = \begin{bmatrix} c\alpha c\beta & (CB)c\alpha s\beta & -s\alpha & 0 & (ZB) & (YB) \\ -(BC)s\alpha c\beta s\phi & -s\alpha s\beta s\phi & -(BC)c\alpha s\phi & (ZC) & 0 & -(XC) \\ -(BC)s\beta c\phi & +c\beta c\phi & & & & \\ s\alpha c\beta c\phi - s\beta s\phi & (CB)s\alpha s\beta c\phi & c\alpha c\phi & -(YB) & -(XB) & 0 \\ & + (CB)c\beta s\phi & & & & \end{bmatrix} \begin{bmatrix} C_{l_{bal}} \\ C_{m_{bal}} \\ C_{n_{bal}} \\ C_{A_b} \\ C_{Y_b} \\ C_{N_b} \end{bmatrix}$$

Legend

$$(CB) = \frac{\bar{c}}{b} \quad (BC) = \frac{b}{\bar{c}} \quad (ZC) = \frac{Z}{\bar{c}}$$

$$(ZB) = \frac{Z}{b} \quad (YB) = \frac{Y}{b} \quad (XC) = \frac{X}{\bar{c}} \quad (XB) = \frac{X}{b}$$

$$b = \text{span} \quad s = \sin \quad c = \cos$$

$$\bar{c} = \text{M.A.C} \quad X, Y, Z = \text{body transfer distances}$$

Generally, roll ( $\phi$ ) is considered as zero in the balance to body transformation.

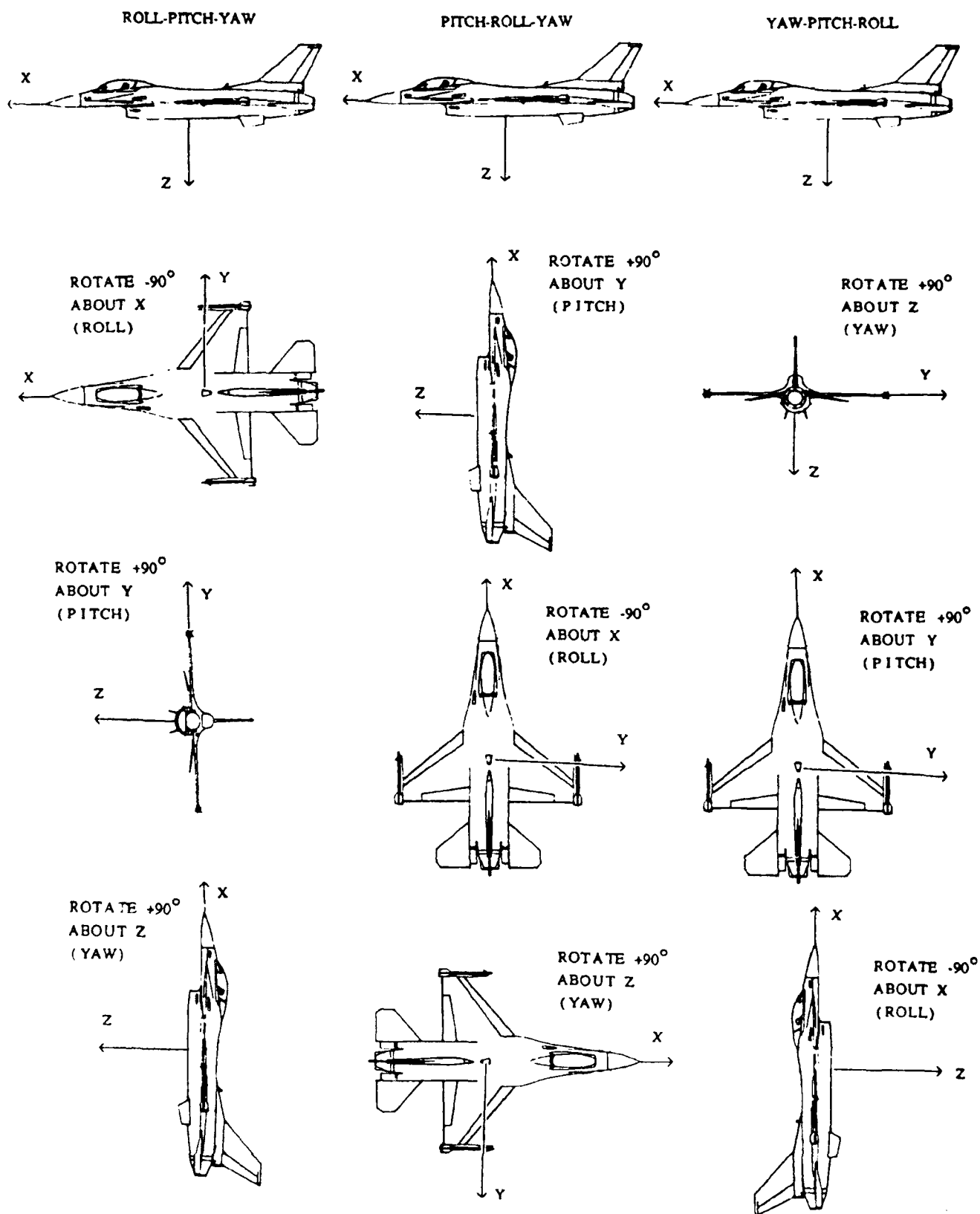


Figure IV-5 Rotation Order



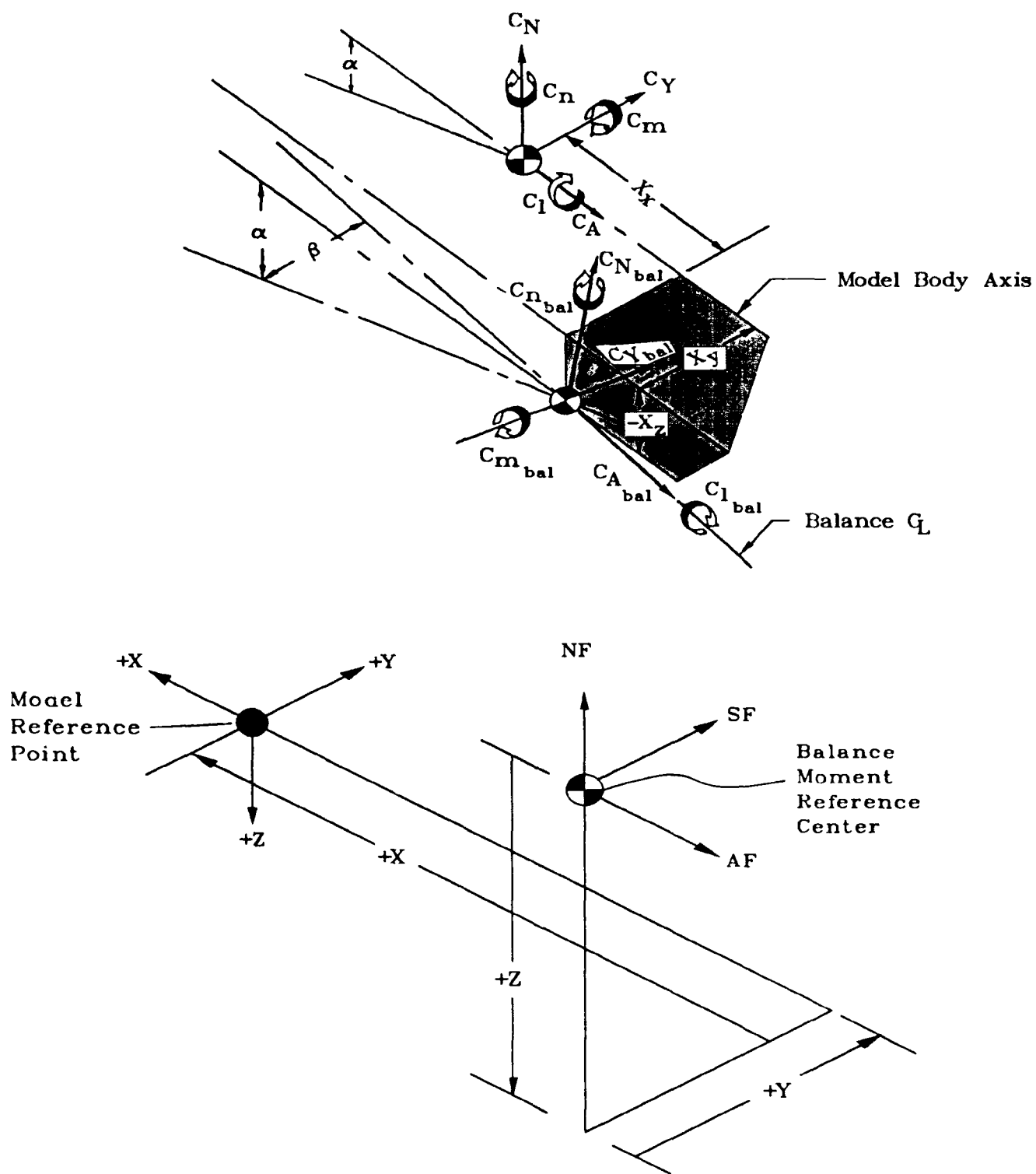


Figure IV-6 Balance Axis

### Body Axis to Wind Axis (ref. 4 & 8)

Note:  $\alpha$  and  $\phi$  are aerodynamic angles

$(\bar{c}/B)$  or  $(B/\bar{c})$  is used to add moment coefficients together (applies to apples).

#### Forces

$$\begin{aligned}C_{L_w} &= C_{N_b} \cos \alpha - C_{A_b} \sin \alpha \\C_{D_w} &= C_{A_b} \cos \alpha \cos \phi + C_{Y_b} \sin \phi + C_{N_b} \sin \alpha \cos \phi \\C_{Y_w} &= C_{Y_b} \cos \phi - C_{A_b} \cos \alpha \sin \phi - C_{N_b} \sin \alpha \sin \phi\end{aligned}$$

#### Forces

$$\begin{bmatrix} C_{L_w} \\ C_{D_w} \\ C_{Y_w} \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha \cos \phi & \cos \alpha \cos \phi & \sin \phi \\ -\sin \alpha \sin \phi & -\cos \alpha \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} C_{N_b} \\ C_{A_b} \\ C_{S_b} \end{bmatrix}$$

#### Moments

$$\begin{aligned}C_{m_w} &= C_{m_b} \cos \phi + (B/\bar{c}) C_{l_b} \cos \alpha \sin \phi + (B/\bar{c}) C_{n_b} \sin \alpha \sin \phi \\C_{l_w} &= C_{l_b} \cos \alpha \cos \phi - (\bar{c}/B) C_{m_b} \sin \phi + C_{n_b} \sin \alpha \cos \phi \\C_{n_w} &= C_{n_b} \cos \alpha - C_{l_b} \sin \alpha\end{aligned}$$

#### Moment

$$\begin{bmatrix} C_{m_w} \\ C_{l_w} \\ C_{n_w} \end{bmatrix} = \begin{bmatrix} \cos \phi & (B/\bar{c}) \cos \alpha \sin \phi & (B/\bar{c}) \sin \alpha \sin \phi \\ -(\bar{c}/B) \sin \phi & \cos \alpha \cos \phi & \sin \alpha \cos \phi \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} C_{m_b} \\ C_{l_b} \\ C_{n_b} \end{bmatrix}$$

$B$  = wing span  
 $\bar{c}$  = M.A.C.

Body Axis to Stability Axis (ref. 4)

Forces

$$C_{L_s} = C_{N_b} \cos\alpha - C_{A_b} \sin\alpha$$

$$C_{D_s} = C_{N_b} \sin\alpha + C_{A_b} \cos\alpha$$

$$C_{Y_s} = C_{Y_b}$$

Forces

$$\begin{bmatrix} C_{L_s} \\ C_{D_s} \\ C_{Y_s} \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{N_b} \\ C_{A_b} \\ C_{S_b} \end{bmatrix}$$

Moments

$$C_{m_s} = C_{m_b}$$

$$C_{l_s} = C_{l_b} \cos\alpha + C_{n_b} \sin\alpha$$

$$C_{n_s} = C_{n_b} \cos\alpha - C_{l_b} \sin\alpha$$

Moments

$$\begin{bmatrix} C_{m_s} \\ C_{l_s} \\ C_{n_s} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} C_{m_b} \\ C_{l_b} \\ C_{n_b} \end{bmatrix}$$

## Wind Axis to Stability Axis (ref. 4 & 8)

### Forces

$$C_{L_s} = C_{L_w}$$

$$C_{D_s} = C_{D_w} \cos\phi - C_{Y_w} \sin\phi$$

$$C_{Y_s} = C_{Y_w} \cos\phi + C_{D_w} \sin\phi$$

### Force

$$\begin{bmatrix} C_{L_s} \\ C_{D_s} \\ C_{Y_s} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} C_{L_w} \\ C_{D_w} \\ C_{Y_w} \end{bmatrix}$$

### Moments

$$C_{m_s} = C_{m_w} \cos\phi - (B/\bar{c}) C_{l_w} \sin\phi$$

$$C_{l_s} = C_{l_w} \cos\phi + (\bar{c}/B) C_{m_w} \sin\phi$$

$$C_{n_s} = C_{n_w}$$

### Moments

$$\begin{bmatrix} C_{m_s} \\ C_{l_s} \\ C_{n_s} \end{bmatrix} = \begin{bmatrix} \cos\phi & - (B/\bar{c}) \sin\phi & 0 \\ (\bar{c}/B) \sin\phi & \cos\phi & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{m_w} \\ C_{l_w} \\ C_{n_w} \end{bmatrix}$$

$B$  = wing span

$\bar{c}$  = M.A.C

Since all of the fore mentioned coordinate transformation equations are orthogonal matrices (square matrices), one can obtain the 'other' axis transformation by simply transposing the matrix.

## NOTES

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## NOTES

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## NOTES

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## V TRIP STRIPS

## V Trip Strips

### Boundary Layer Symbols

$R_k$	Reynolds number based on roughness height, velocity, and kinematic viscosity at top of roughness ( $V_k k / \nu_k$ )
$R_x$	Reynolds number based on freestream conditions and distance of roughness from leading edge ( $V_\infty x / \nu_\infty$ )
$k$	Roughness height
$x$	Distance of roughness from leading edge
$u$	Local streamwise velocity component outside the boundary layer
$u_k$	Local streamwise velocity component inside the boundary layer at the top of the roughness particle
$V_\infty$	Freestream velocity
$x$	Distance from leading edge to roughness strip (trip strip)
$\delta$	Boundary layer thickness
$\nu_k$	Kinematic viscosity at top of the roughness particle
$\nu_\infty$	Freestream kinematic viscosity
$T_t$	Total temperature ( $^{\circ}\text{F}$ )

### Boundary Layer Discussion (ref. 7)

Due to the effects of viscosity, the velocity near a surface is gradually slowed from the freestream velocity,  $V_\infty$ , to zero velocity. The region where this velocity change occurs is called the boundary layer. A boundary layer in which the velocity varies approximately linearly from the surface is called laminar and a boundary layer whose velocity varies approximately exponentially from the surface is called turbulent. Generally speaking, the upper Reynolds number (Rn) limit for a laminar boundary layer is  $1 \times 10^6$  per ft. However, transition from a laminar to a turbulent boundary layer most often occurs at a much lower Reynolds number ( $5 \times 10^5$  per ft).

### Boundary Layer Thickness

The boundary layer thickness is defined as the distance from the surface to a point where the velocity in the boundary layer is 99% of the velocity of  $V_\infty$ . The boundary layer thickness can be approximated by

#### Laminar

$$\delta = 5.2 (l^2/Rn)^{0.5}$$

#### Turbulent

$$\delta = 0.37 l/(Rn)^{0.2}$$

where

$l$  = distance from body leading edge

$Rn$  = Reynolds number =  $(\rho V_\infty l / \mu)$

$\rho$  = air density \* 32.1741 (ft/sec<sup>2</sup>)

$V_\infty$  = freestream velocity

$l$  = some reference length

$\mu$  = absolute viscosity

=  $(1.2024 \times 10^{-5} \text{ lbm/ft-sec})$

=  $(3.7373 \times 10^{-7} \text{ slugs/ft})$

### Trip Strips (ref. 8)

A trip strip is an artificial roughness added to the model to fix the location of transition from laminar to a turbulent boundary layer. The reason a trip strip is added to a wind tunnel model is to increase the local effective Reynolds number and to "duplicate" the boundary layer to that of a full scale test article. Some general guidelines that are applicable to all grit-type trips are listed below (ref. 25):

- 1) The roughness bands should be narrow (0.125 to 0.25 inches).
- 2) The roughness should be sparsely distributed. According to reference 8, approximately one grain per every 2mm (.080 in) along the trip strip is the closest desirable spacing; grains could possibly be spaced as far as 5mm (.200 in) apart and still cause transition. However, if the trip strip is either too high above the surface or is too densely packed with particles, it can affect the model drag, maximum lift and not fix transition.
- 3) Two-dimensional trips are unsatisfactory because reasonable heights do not fix transition at the trip location.
- 4) Care should be taken not to build up layers of adhesive which can form spanwise ridges at the edge of the trip. These ridges also tend to make the trip act as a two-dimensional step.

### Trip Strip Types (ref. 8)

#### Grit

The traditional trip strip is a finite width strip of grit. Two commercially available grit materials that are used are Carborundum and Ballotine micro beads or balls. Trip strip width is usually 0.100 to 0.25 inches. Clear lacquer or double sided tape can be used as a gluing agent.

#### Two-Dimensional Tape

These consist of 0.125 inch height printed circuit drafting tape or chart tape. Also, cellophane type tape can be used. The trip strip is built up by multiple layers of tape.

### Epoxy Dots

A vinyl tape of varying thicknesses that has holes of 0.05 inch diameter and 0.10 inch center to center displacement is used. This tape is applied to the model surface and an epoxy compound is forced into the holes. Once the epoxy has hardened remove the tape.

### Thread or String

Thread or string is glued to the model. This technique is not used much any more.

### Location of Trip Strips

#### Lifting Surfaces

This includes wings, tails, and fins. The trip strip is applied to both sides of the lifting surface. For four and five digit airfoils and conventional wing constructions, the full scale transition will occur approximately 10% of the chord at cruise conditions.

#### Fuselage

The trip strip is often located where the local diameter is one-half of the maximum diameter. Care should be taken to ensure that the flow has not reattached or that laminar flow has been reestablished aft of the trip strip.

#### Nacelles

For flow through nacelles, the trip strip is placed inside the nacelle and is located approximately 5% aft of the inlet L.E. Also, additional trip strip is located on the outside at 5-10% aft of the inlet L.E.

From reference 8, testing of models through a range of Reynolds and Mach numbers, it is desirable to eliminate the need for changing grit sizes for each condition. This elimination is accomplished by using a grit (roughness) size determined for the



combination of test Reynolds number and Mach number which require the largest grit size - usually at the smallest Reynolds number and largest Mach number condition. For tapered wings (ref. 10), apply grit at a constant percentage of the local chord (nominally 5%). In order to permit the use of a single grit size across the span, the grit size is calculated for the largest chord. For wings with a sharp supersonic leading edge, calculate the grit size using the Mach number and unit Reynolds number based on the flow outside the boundary layer on the upper surface at the maximum test angle of attack. Apply grit to both upper and lower wing surfaces. For sharp subsonic leading-edge wing or round leading-edge wing, whether or not swept behind the Mach line, the freestream Mach number and unit Reynolds number are used to determine the grit size. Using the above criteria, there will be test conditions when grit particles are larger than the minimum required to cause transition. However, careful application of a sparse distribution of grit particles on a narrow strip will minimize the drag contribution of the grit itself.

#### Determination of Trip Strip Height

Grit height determination is a black art. The testing engineer, having an understanding of boundary layer build-up/profile (at low  $q$ , the boundary layer height is larger than at high  $q$ ), can make a determination of the grit height based on a proven method. Also, don't forget the helpful hints from the "old guard" that has determined grit height many times previously. The correct method in determining the grit height for establishing boundary layer transition is to accomplish a drag study based on grit height. But reality (money and time) dictates the test and the use of reference 25 is generally adequate. If the test has variable dynamic pressure ( $q$ ) runs, the proper method is to have a new grit height at each  $q$ . However, economics (\$) will dictate if a grit study will be accomplished.

Two methods of determining boundary layer and grit heights based on experimental data are presented. The first method is for atmospheric tunnels only. The second method can be used for either atmospheric or pressure tunnels.

#### Method 1 (Atmospheric tunnels; ref. 11)

This method of determining boundary layer and grit height is for wind tunnels whose test section is roughly at one (1) atmosphere. In other words, there is no control over the total pressure or total temperature and consequently Reynolds number per foot is a

function of velocity or Mach number. This method is based upon reference 11 (flat plate) and each figure (figures V-1 thru V-3) is represented by two total temperature curves,  $T_t = 40^\circ\text{F}$  and  $T_t = 120^\circ\text{F}$ .

The effect of increasing the temperature is to increase the boundary layer thickness at each Mach number. Also by increasing the Mach number (Reynolds number per foot), the boundary layer thickness decreases at any station (on a flat plate).

It is desirable to prevent the transition point from moving on the model during the test. By applying the correct grit at the correct distance, the boundary layer will transition from laminar to turbulent near the grit grains at a fixed position. The conditions which these grains cause transition and remain fixed at subsonic speed, are generally dependent upon the length Reynolds number,  $R_x$ , and the height Reynolds number,  $R_k$ .

Reference 25 (using ref. 11 as a reference) points out not to try to cause artificial transition below a length Reynolds number of approximately 100,000. The first restriction is line "A" on figure V-1. This restriction has the practical effect of moving the transition strip aft of the wing leading edge to a fixed dimension rather than a fixed percentage of the chord. Using the lowest Mach number on figure V-1, the minimum distance is approximately one (1) inch. Another restriction based upon the length Reynolds number,  $R_x$ , is the location on the surface where transition from laminar to turbulent first starts to occur naturally. Natural transition occurs on a flat plate at an approximate length Reynolds number of 680,000 and the corresponding distance is represented by line "B" on figure V-1. Since transition starts naturally at line "B", it's desirable to place the trip strip ahead of "B" at a given set of test conditions. This action precludes the possibility of the natural transition point moving ahead of the trip strip which could possibly be attributed to an adverse pressure gradient.

A curve of grit height verse Mach number is shown in figure V-3. Also shown is a curve of boundary layer height at a length Reynolds number  $R_x = 100,000$ . The critical grit height is shown to be well within the boundary layer at each Mach number. Reference 25 has shown that transition may be achieved by the range of grit size above the minimum. However, when the grit protrudes from the boundary layer, measurable drag is created. Satisfactory artificial transition of boundary layer from laminar to turbulent may be caused at a given Mach number by grit sizes which lie between the two curves in figure V-3.

Shown in figure V-4 is a curve of Grit height verses Carborundum Grit Number. This figure can be used to determine the use of nominal grit sizes that will provide enough grains to cause transition provided the spacing of the individual grains is correct.

In order to satisfy the condition for minimum influence of the particles, some consideration must be given to particle density and to the width of the grit strip. It has been demonstrated experimentally in reference 25 that approximately one grit per every 2mm (.080 in) along the transition line is the closest desirable spacing; grains could possibly be spaced as far as 5mm (.200 in) apart and still cause transition (sublimation chemicals can help determine the transition point). The width of the trip strip should be on the order of 1mm (0.04 in).

The maximum forward location of the trip strip required to cause transition is shown in figure V-2 at two (2) different total temperatures. Figures V-2 and V-3 are used to determine the trip strip grain size and trip location.

#### Method 2 (Atmospheric or Pressure tunnels)

This method for determining the height of the trip strip originated from reference 10. Figure V-5 is derived from reference 10 and is included in this report.

Below is an example using Method 2 to determine the grit height required to start transition and the corresponding carborundum grit number.

#### Example

Wing (2-D curves)

$X = 0.5$  inches (position of grit from LE)

$M = 3.0$

$RN/FT = 4 \times 10^6$

$$X * \frac{R_{\infty} / ft}{1 \times 10^6} = X * \frac{4 \times 10^6}{1 \times 10^6} = 4X$$

$$X * \frac{R_{\infty} / ft}{1 \times 10^6} = (.5)(4) = 2 \text{ inches}$$

From figure V-5 (2-D, Mach 3.0) at 2.0

$$K * \frac{R_{\infty} / ft}{1 \times 10^6} = 0.0264$$

$$K * \frac{4 \times 10^6}{1 \times 10^6} = 4K = 0.0264$$

$$K = 0.0066 \text{ inches}$$

From figure V-3 or table V-1 at  $K = 0.0066$  a grit of 90 is to be used.

Table V-1  
Nominal Grit Size

Grit Number	Nominal Grit Size (in)
10	0.0937
12	0.0787
14	0.0661
16	0.0555
20	0.0469
24	0.0331
30	0.0280
36	0.0232
46	0.0165
54	0.0138
60	0.0117
70	0.0098
80	0.0083
90	0.0070
100	0.0059
120	0.0049
150	0.0041
180	0.0035
220	0.0029
320	0.0017

#### Application of Grit types of Trip Strips (ref. 8)

Masking tape is typically used to lay out the trip strips (at a certain width apart). Then shellac, lacquer, artist's clear acrylic, or even superhold hair spray is painted or sprayed over the trip strip area. Once the adhesive material has covered the surface, the grit material is dusted or blown on the wet adhesive. Grit usually is difficult to apply

to vertical and lower surfaces. To aid in applying grit to those surfaces, a piece of paper or cardboard can be shaped into a 'V' and with skill can be blown onto the surface. If the grit is too densely packed, use a tooth-pick to pick off selected grit particles.

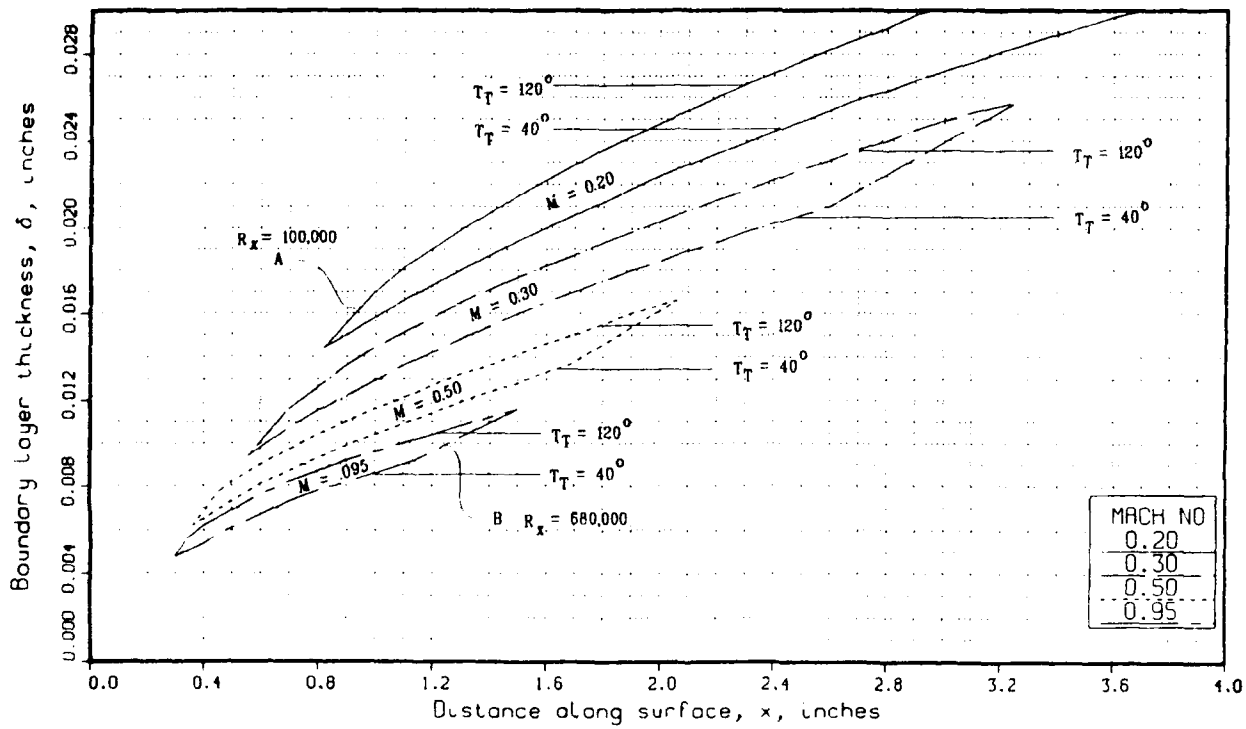


Figure V-1 Boundary Layer Thickness

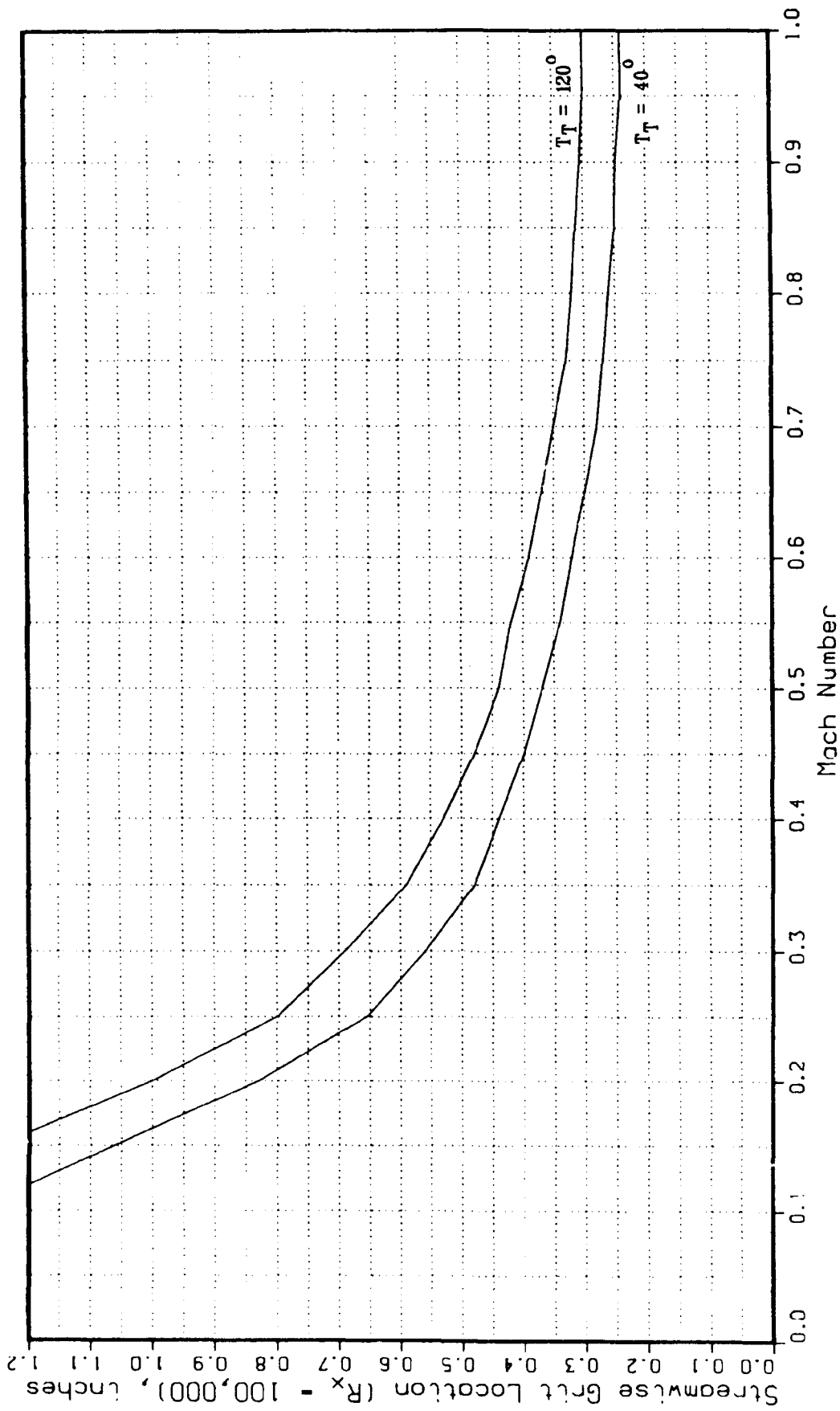


Figure V-2 Streamwise Grit Location

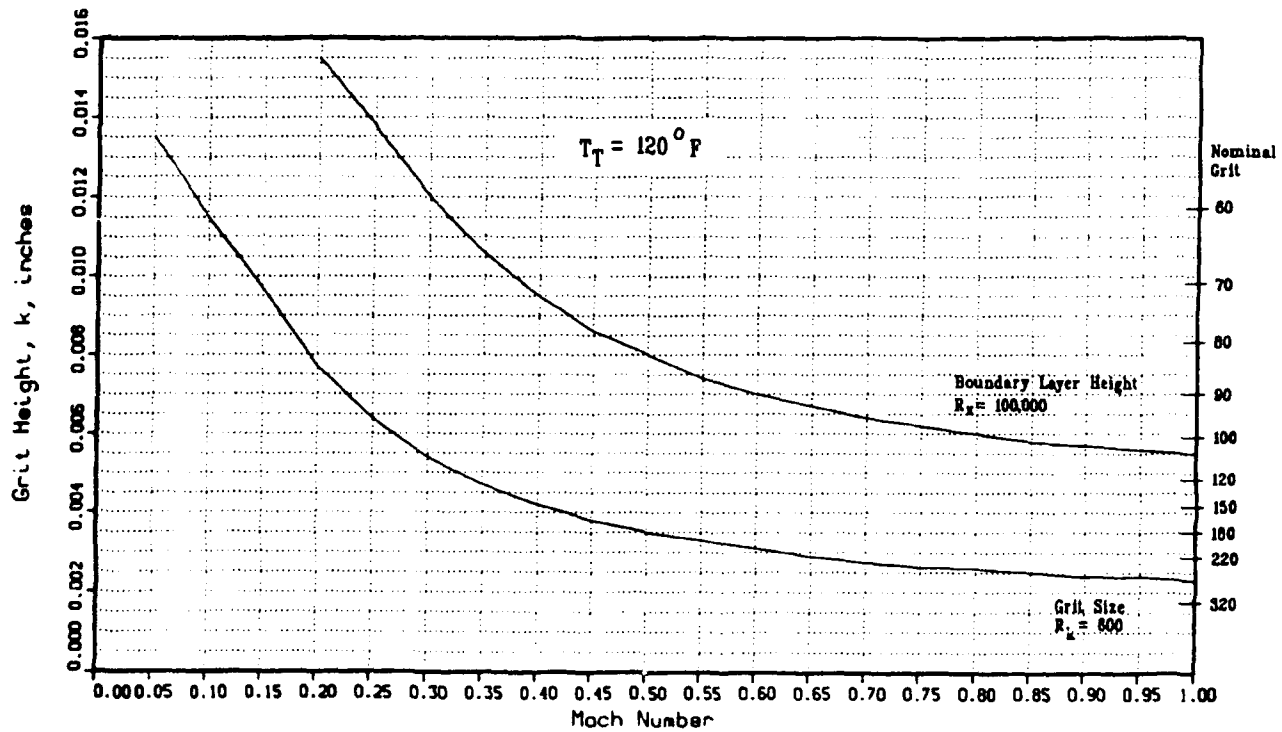


Figure V-3 Grit Height



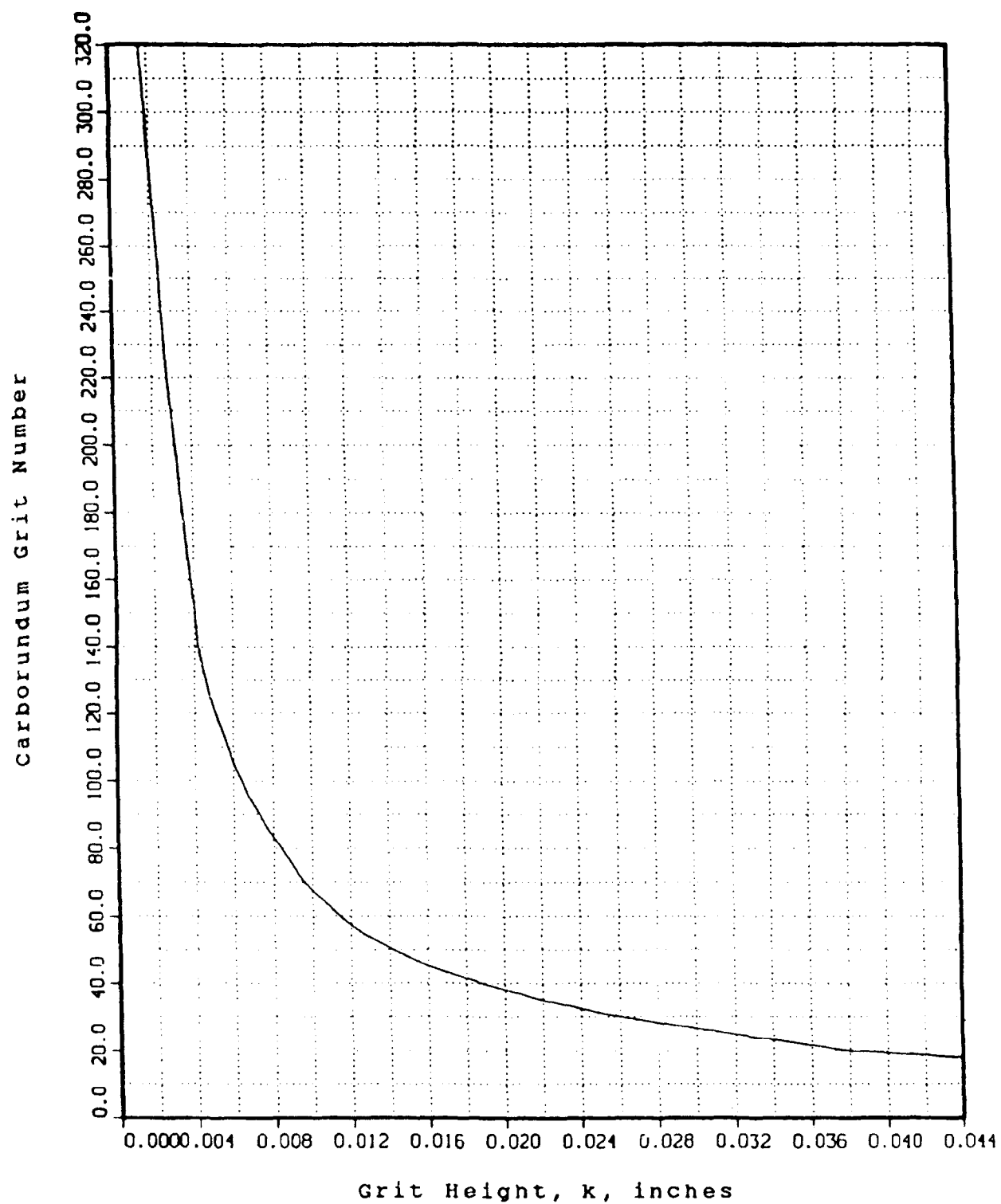


Figure V-4 Carborundum Grit Number

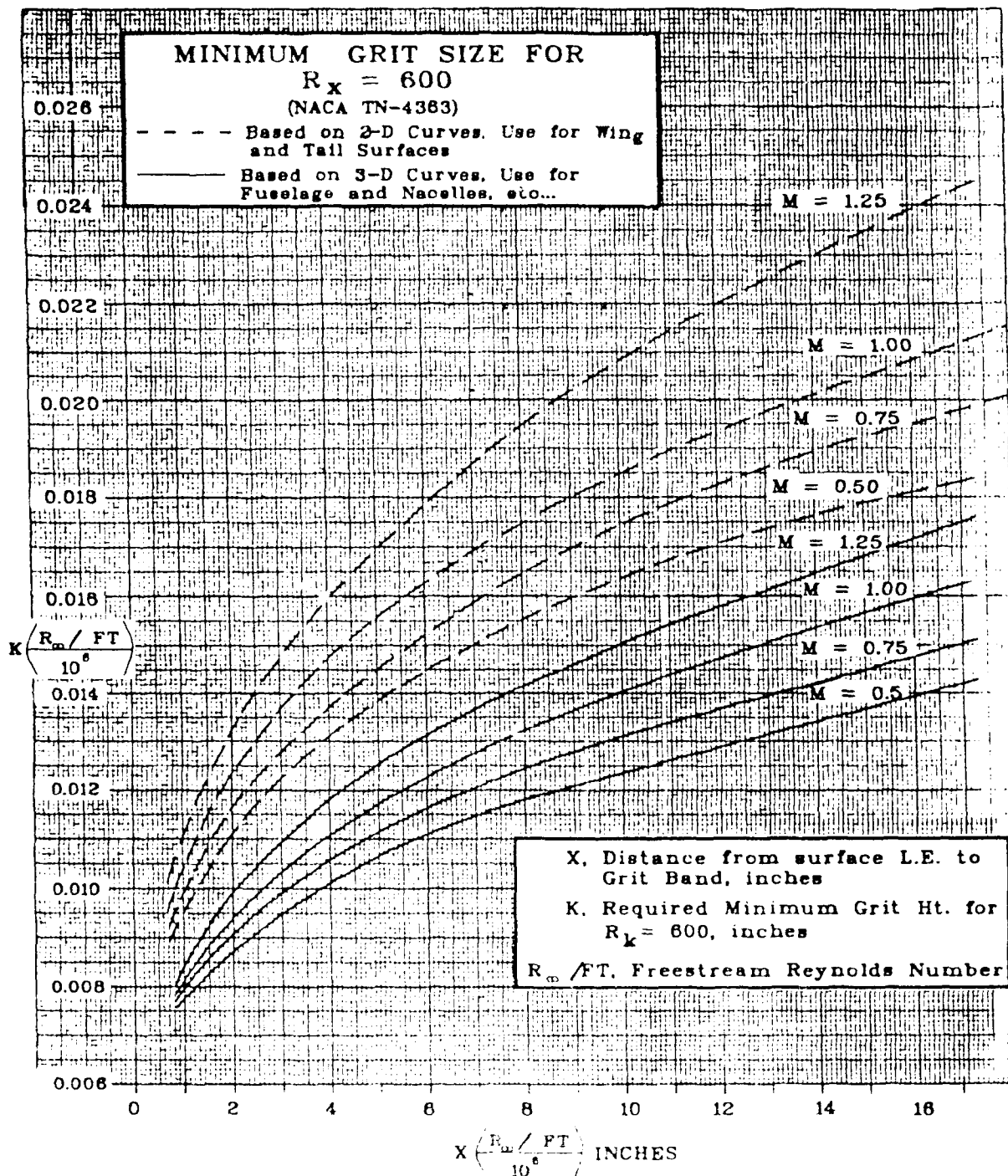


Figure V-5  
 Minimum Grit Size  
 (Method 2)

**Example**

Wing with  $X = 1.125$

$M_\infty = 0.75$ ;  $R_\infty = 8 \times 10^6$  / ft

$$X \left( \frac{R_\infty}{1 \times 10^6} \right) = \left( \frac{8 \times 10^6}{1 \times 10^6} \right) = 8X$$

$$(1.125)(8) = 9.125 \text{ inches}$$

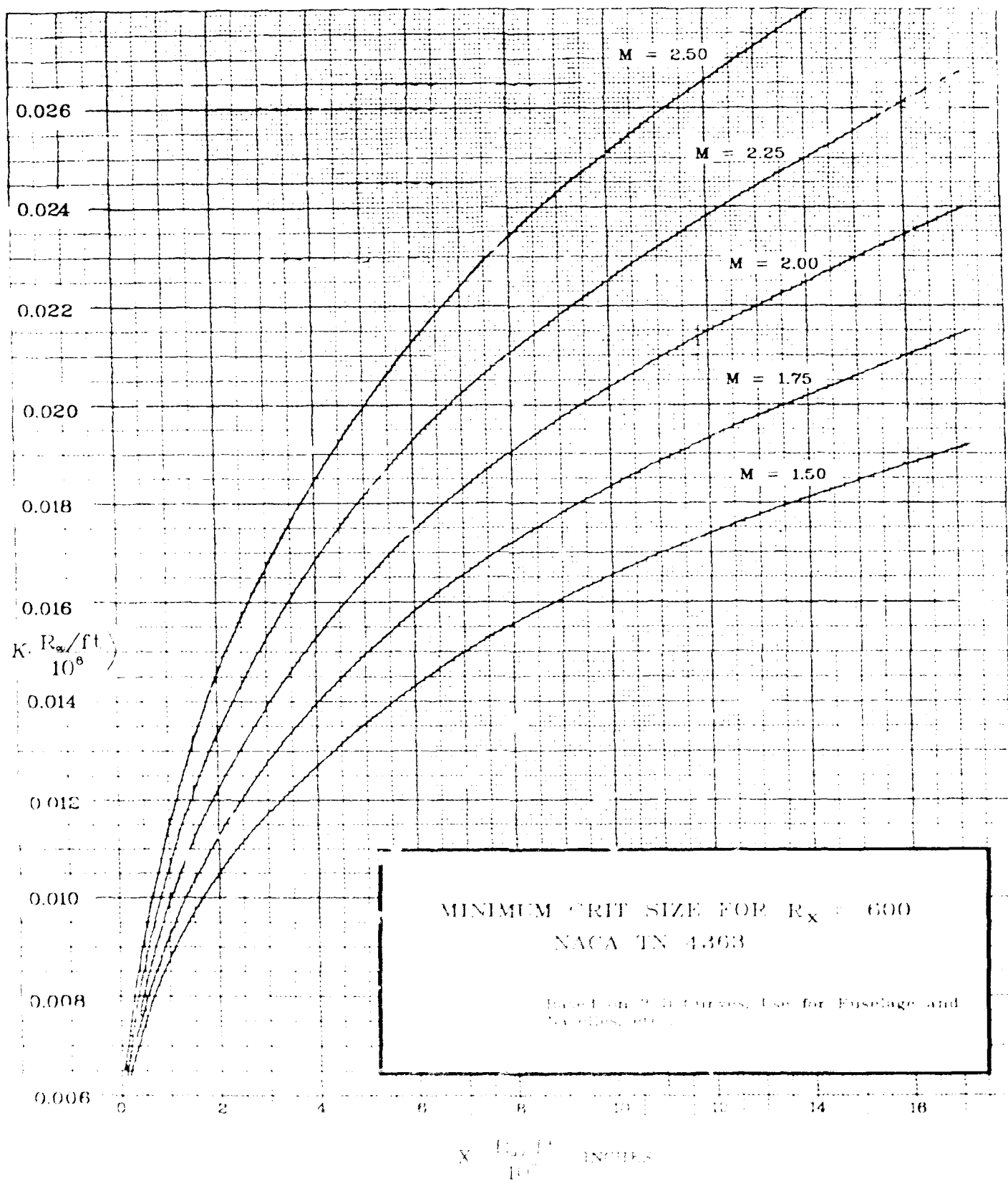
From above chart at 8.75

$M_\infty = 0.75$ ; 2D

$$K \left( \frac{R_\infty / ft}{10^6} \right) = 0.0157$$

$$8K = 0.0157$$

$$K = 0.0020$$



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Page 1

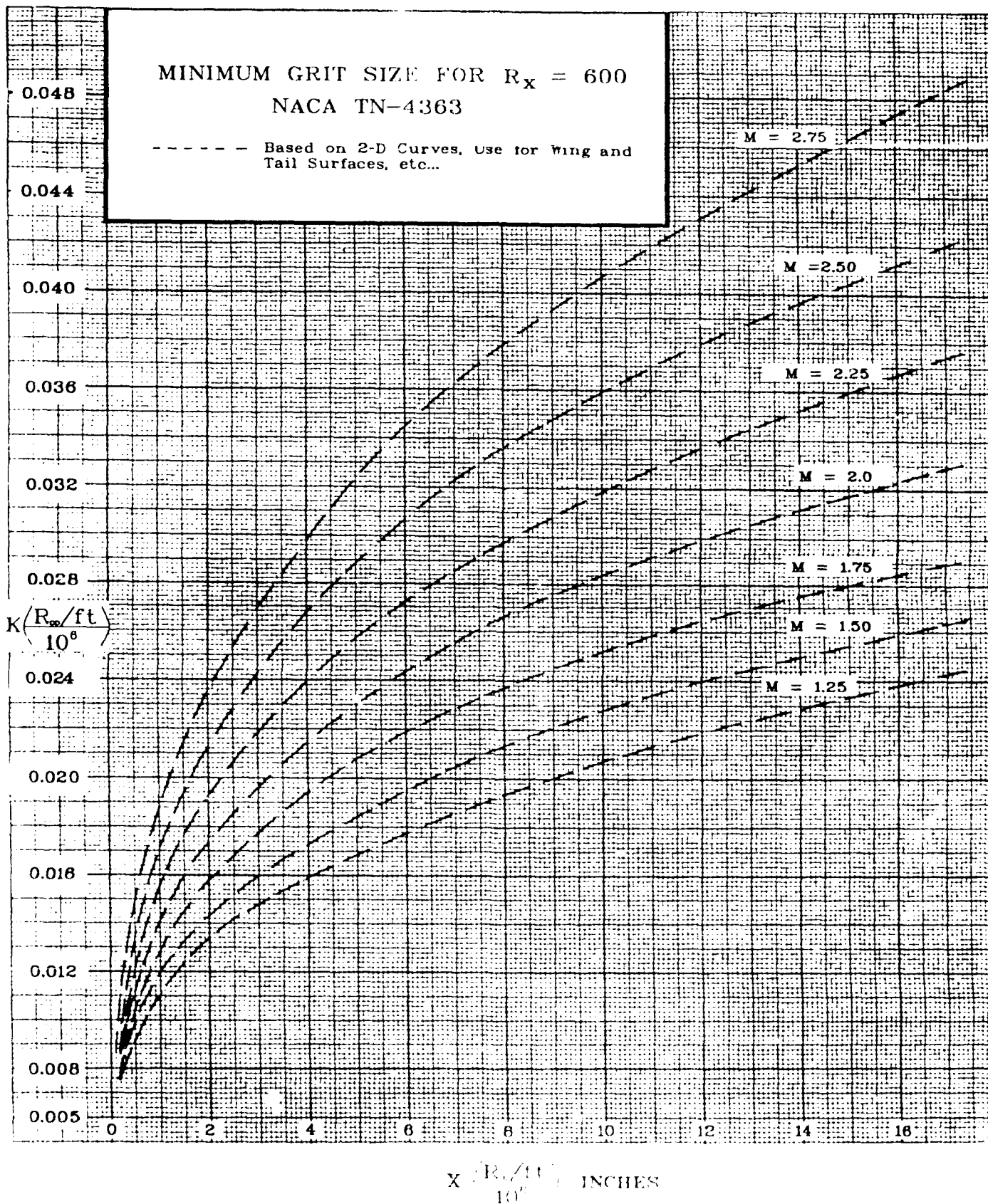


Figure 7-3 continued

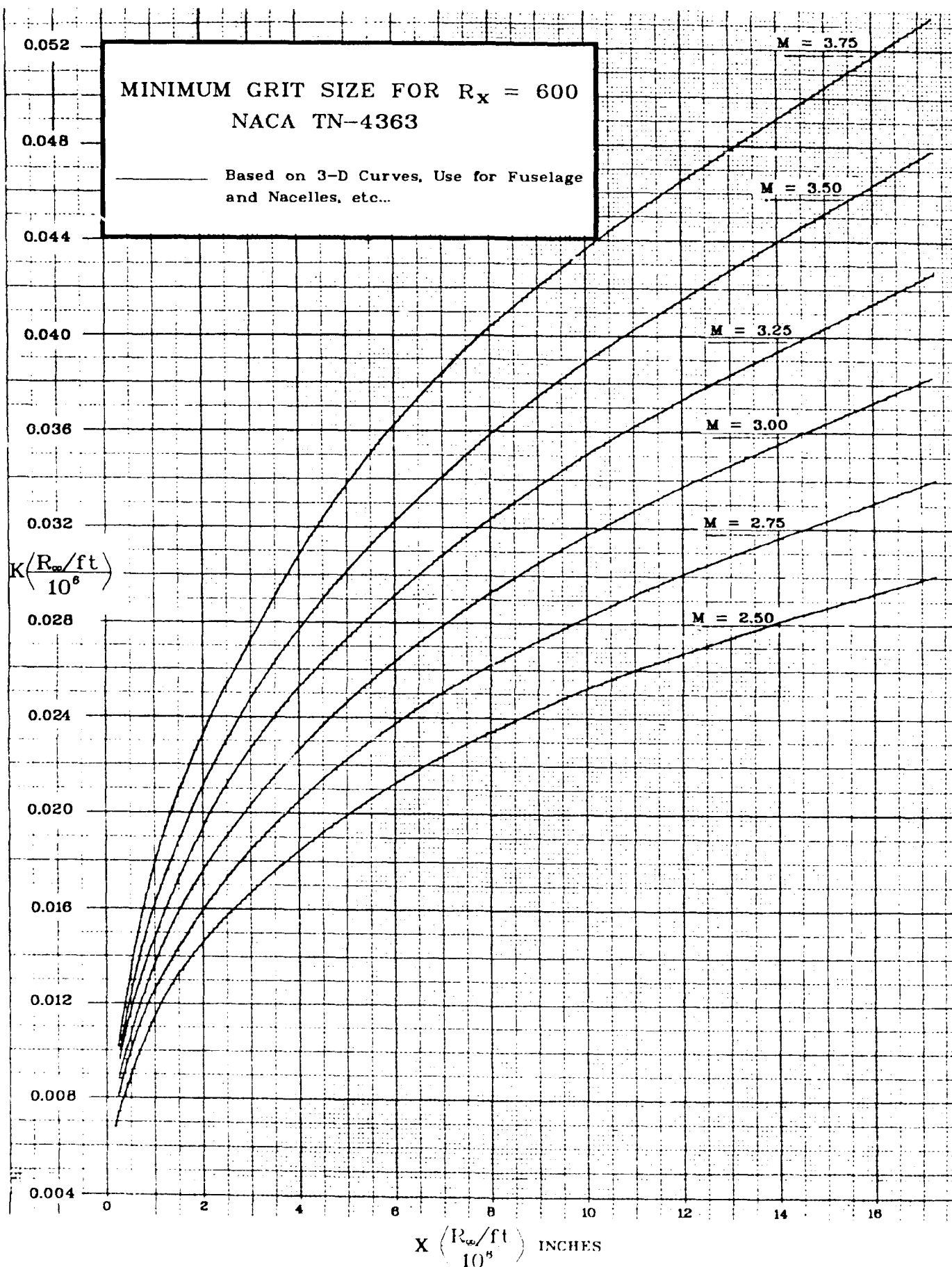


Figure V-5 continued

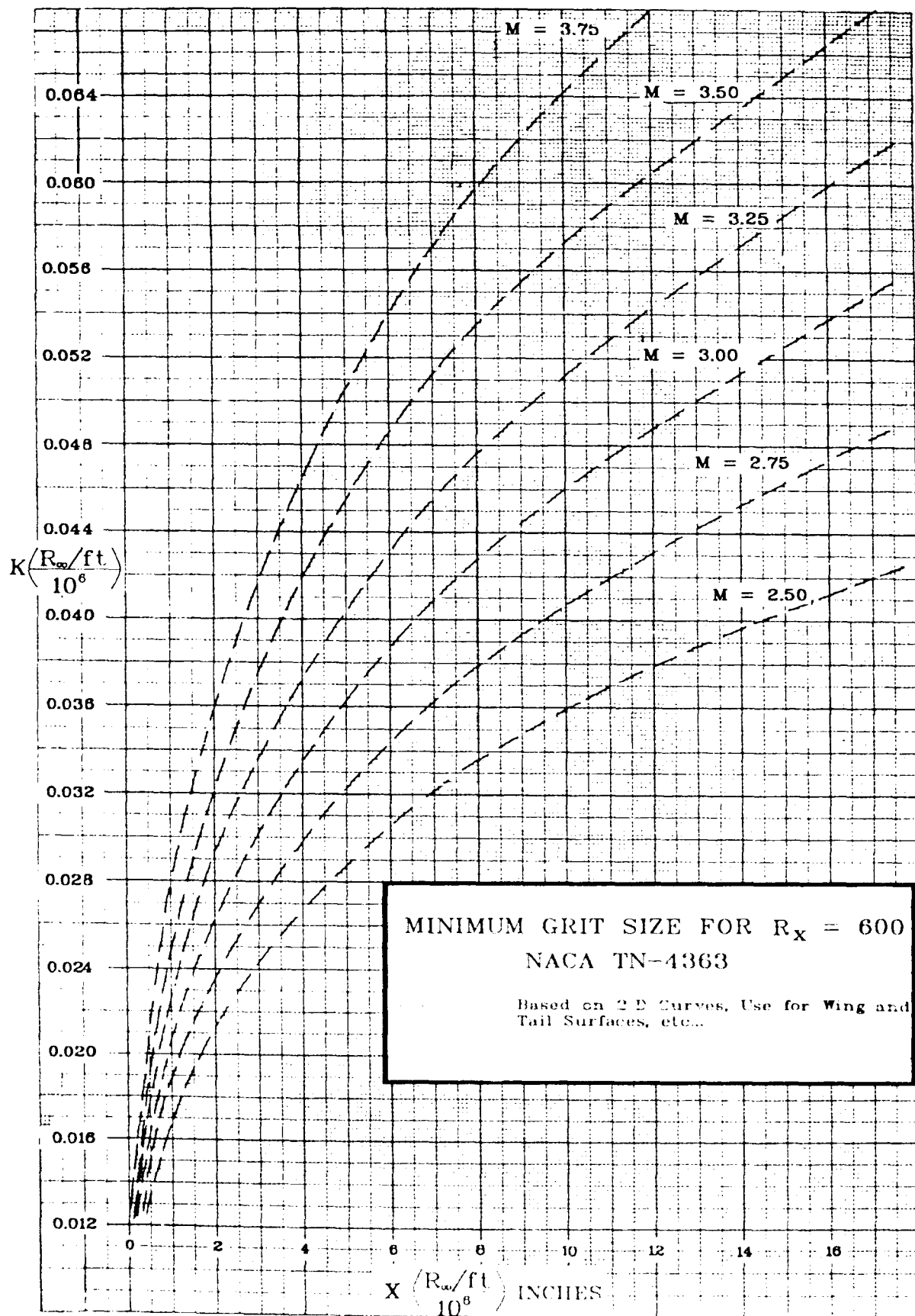


Figure V-5 concluded

## NOTES

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## VI PLANFORM CHARACTERISTICS

## VI Planform Characteristics

### Planform Symbols

AR	Aspect ratio
a	Cutout factor
b	Full wing span
b/2	Half wing span
b/(2l)	Wing-slenderness parameter
$b_i$	Span of inboard planform formed by two panels
$b_o$	Span of outboard planform formed by two panels
c	Chord (parallel to axis of symmetry) at any given span station y
$\bar{c}$	Mean Aerodynamic Chord (MAC)
$c_B$	Chord at span break station
$c_r$ or $C_r$	Root chord
$c_t$ or $C_t$	Tip chord
l	Over-all length from wing apex to most aft point on the trailing edge
m,n	Non-dimensional chordwise stations in terms of c
p	Planform-shape parameter
S	Wing area
$S_i$	Total area of inboard panels
$S_o$	Total area of outboard panels
$S_W$	Wing area affected by trailing edge deflection
$\Delta S$	Incremental wing area
x	Chordwise location of leading edge at span station y
$x_{\text{centroid}}$	Chordwise location of centroid of area (chordwise distance from apex to c/2)
$x_{\text{MAC}}$ or $\bar{x}$	Chordwise location of mean aerodynamic chord
$y_{\text{MAC}}$ or $\bar{y}$	Spanwise location of MAC (equivalent to spanwise location of centroid of area)
$\lambda$	Taper ratio
$\Lambda_{\text{LE}}$	Sweep angle of leading edge
$\Lambda_{\text{TE}}$	Sweep angle of trailing edge
$\Lambda_n, \Lambda_m$	Sweep angles of arbitrary non-dimensional chordwise locations
$\eta$	Non-dimensional span station

$\alpha$	Angle-of-attack
$\eta_i, \eta_o$	Non-dimensional span stations at inboard and outboard edges of control, respectively
$\eta_B$	Non-dimensional spanwise location of leading edge break in wing
$\sigma$	Ratio of chordwise position of leading edge at tip to root chord length
$\zeta_B$	Chordwise location of break in leading edge sweep(s) in terms of chordwise distance to leading edge at tip = $x_B/x_t$

#### Subscripts

B	Refers to span station where leading/trailing edge change sweep angle
MAC	Mean aerodynamic chord
i,o	Inboard and outboard panels respectively
ZL	Zero lift

## Wing Parameters Definitions (figure VI-1)

Chord Line     A straight line between the leading edge and the trailing edge of the airfoil in the streamwise direction

Mean Camber Line     A line described by the points which are equidistant from the upper and lower wing surfaces.

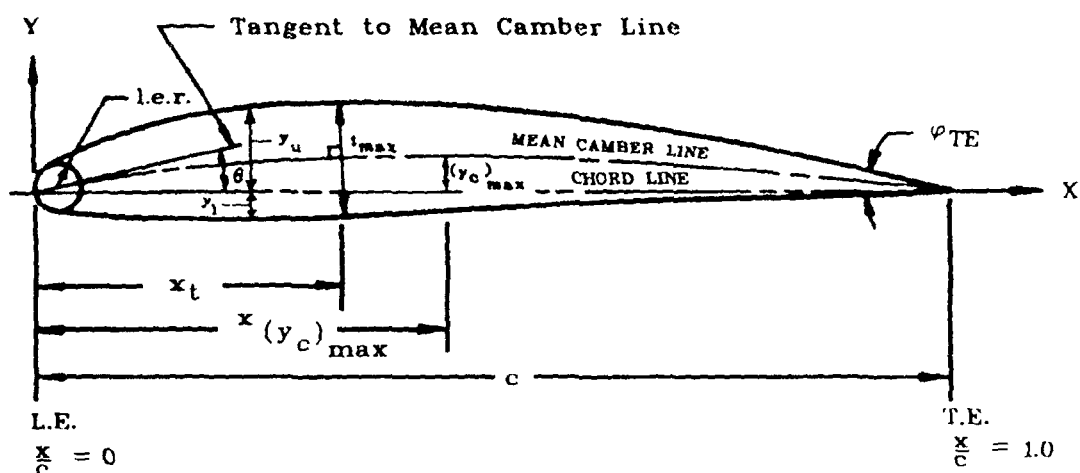
Camber     Is a measure of curvature of an airfoil and is measured as the maximum distance between the mean camber line and the chord line and is measured perpendicular to the chord line. Camber is typically measured in % chord. Positive camber is when the camber line is above the chord line and negative when the camber line is below the chord line.

Angle-of-Attack     Angle between the relative wind ( $V_{\infty}$ ) and the chord line.

Zero Lift Line     A line parallel to the relative wind ( $V_{\infty}$ ) and passing through the trailing edge of the airfoil when the airfoil is at zero lift.

Effective Angle of Attack     The angle of attack of the zero lift line measured from the the relative relative wind.

$\alpha_{ZL}$      Approximately equal to the amount of camber in percent chord for airfoils and untwisted wings with constant section.



$X$  = distance along the chord measured from L.E.

$Y$  = ordinate at some  $X$

$c$  = chord

l.e.r. = leading-edge radius

$x(y_c)_{\max}$  = position of maximum camber

$(y_c)_{\max}$  = maximum ordinate of mean camber line

$t_{\max}$  = maximum thickness

$y_u$  = ordinate of upper surface

$y_l$  = ordinate lower surface

$\frac{x}{c}$  = non-dimensionalized chord length

$\theta$  = slope of l.e.r

$\phi_{TE}$  = trailing-edge angle

$x_t$  = position of maximum thickness

(reference 12)

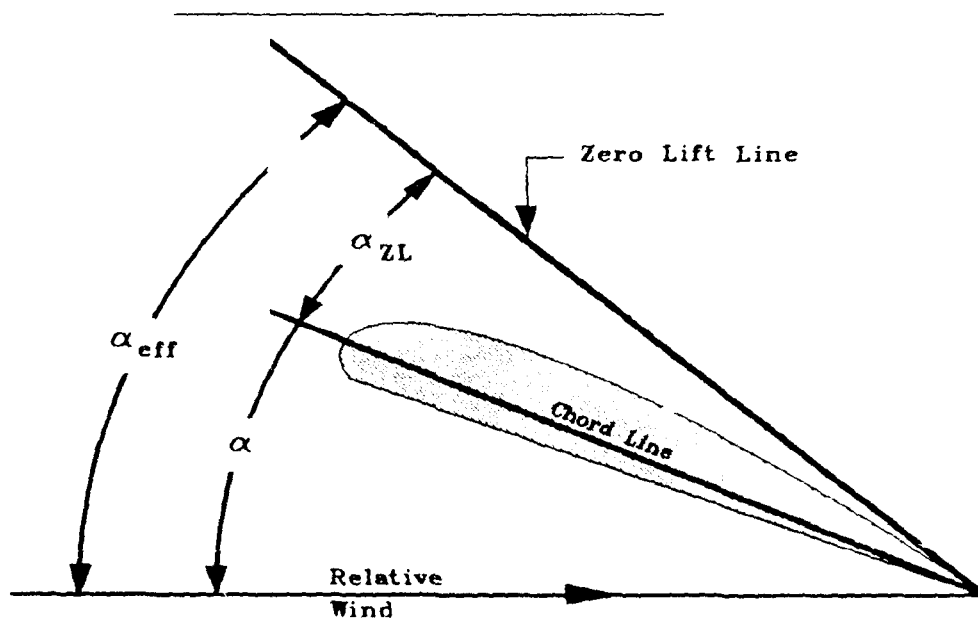


Figure VI-1 Airfoil Nomenclature

## Planform Parameters

### General Planforms

(ref. 12 & 14)

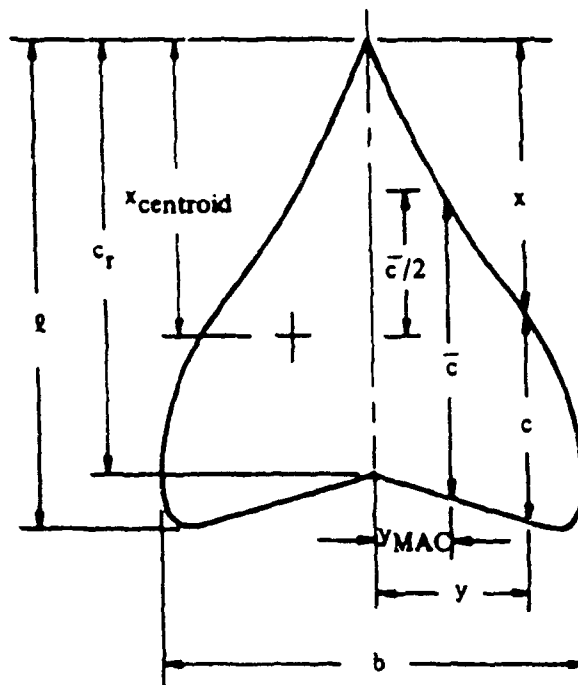


Figure VI-2 General Planform Parameters

$$S = 2 \int_0^{b/2} c \, dy$$

$$\bar{c} = \frac{2}{S} \int_0^{b/2} c^2 \, dy$$

$$\bar{x} = \frac{2}{S} \int_0^{b/2} cx \, dx$$

$$\bar{y} = \frac{2}{S} \int_0^{b/2} cy \, dy$$

$$\bar{x}_{LE} = \bar{x} - \frac{\bar{c}}{2}$$

$$x_{\text{centroid}} = \frac{2}{S} \int_0^{b/2} c(x + \frac{c}{2}) \, dy$$

$$\eta = \frac{(2y)}{B}; \quad p = \frac{S}{(b\Gamma)}; \quad \lambda = C_t / C_r$$

$$\left[ \frac{t}{c} \right]_{\text{RMS}} = \sqrt{\frac{1}{(b/2) - \bar{c}_r} \int_{\bar{c}_r}^{b/2} \left[ \frac{t}{c} \right]^2 \, dy}$$



# Conventional, Straight-Tapered

(ref. 12 & 14)

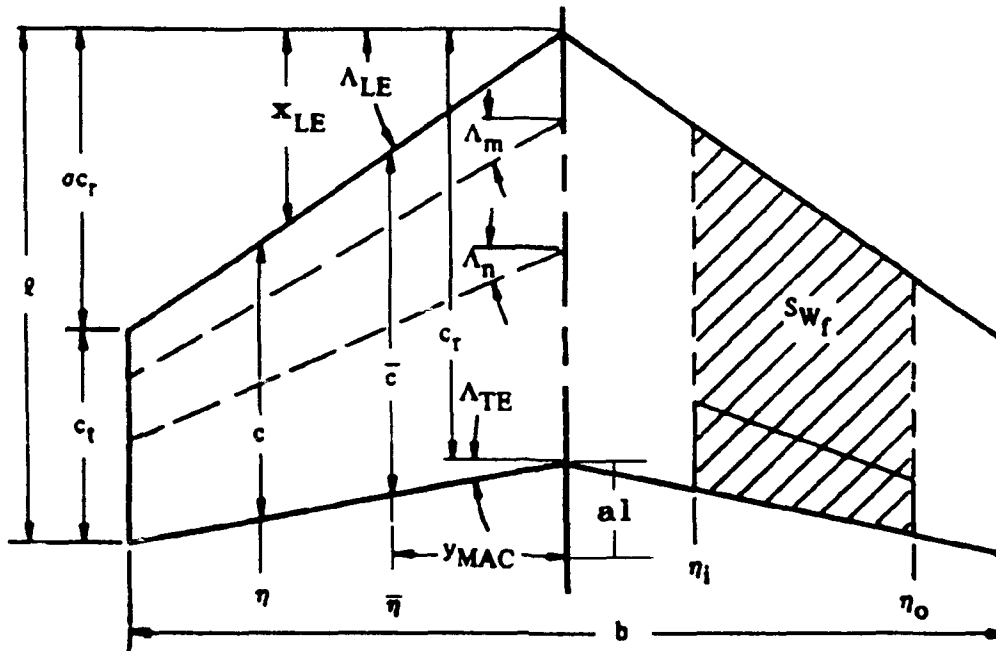


Figure VI-3 Conventional, Straight Tapered Planform

$$AR = \frac{b^2}{S} = \frac{2b}{C_r(1 + \lambda)} = \frac{4(1 - \lambda)}{(1 - a)(1 + \lambda)\tan\Lambda_{LE}}$$

$$S = (b/2)C_r(1 + \lambda) = \frac{b^2}{AR}$$

$$\Delta S = \frac{b}{2} C_r [2 - (1 - \lambda)(\eta_1 + \eta_2)]$$

$$\bar{c} = \frac{2}{3} C_r \frac{1 + \lambda + \lambda^2}{1 + \lambda}$$

$$\bar{y} = \frac{b}{2} \left[ \frac{1}{3} \left[ \frac{1 + 2\lambda}{1 + \lambda} \right] \right]$$

$$\bar{x}_{LE} = \left[ \frac{b}{2C_r} \right] \eta \tan \Lambda_{LE} = \bar{y} \tan \Lambda_{LE}$$

$$\bar{\eta} = (2\bar{y})/b = \frac{1}{3} \left[ \frac{1 + 2\lambda}{1 + \lambda} \right]$$

$$x_{\text{centroid}} = \frac{C_r}{3} \left[ \lambda + \sigma + \left[ \frac{1 + \lambda \sigma}{1 + \lambda} \right] \right]$$

$$c = C_r [1 - \eta(1 - \lambda)] = C_r - y \left[ \frac{C_r - C_t}{(b/2)} \right]$$

$$C_r = \frac{S}{(b/2)(1+\lambda)} = \frac{4(b/2)}{AR(1+\lambda)}$$

$$S_{W_f} = b \frac{[\eta_o - \eta_i]}{2} C_r [2 - (1 - \lambda)(\eta_o + \eta_i)]$$

$$a = \frac{\tan \Lambda_{TE}}{\tan \Lambda_{LE}} = 1 - \frac{C_r(1 - \lambda)}{(b/2)\tan \Lambda_{LE}} = 1 - \frac{4(1 - \lambda)}{AR(1 + \lambda)\tan \Lambda_{LE}}$$

$$\eta = (2y)/b$$

$$\tan \Lambda_n = \tan \Lambda_m - \frac{4}{AR} \left[ (n-m) \left[ \frac{1 - \lambda}{1 + \lambda} \right] \right] \quad n > m$$

$$\tan \Lambda_m = \tan \Lambda_{LE} [1 - (1-a)m]$$

$$\tan \Lambda_{LE} = \frac{1}{a} \tan \Lambda_{TE} = \frac{4 \tan \Lambda_{c/4}}{3 + a}$$

$$\tan \Lambda_{TE} = \frac{4}{AR} - \tan \Lambda_{TE} \quad (\lambda = 0)$$

$$\sigma = \frac{AR}{4} (1 + \lambda) \tan \Lambda_{LE} = \left[ \frac{b}{2C_r} \right] \tan \Lambda_{LE}$$

Double Delta  
and  
Cranked Wings  
(ref. 12 & 14)

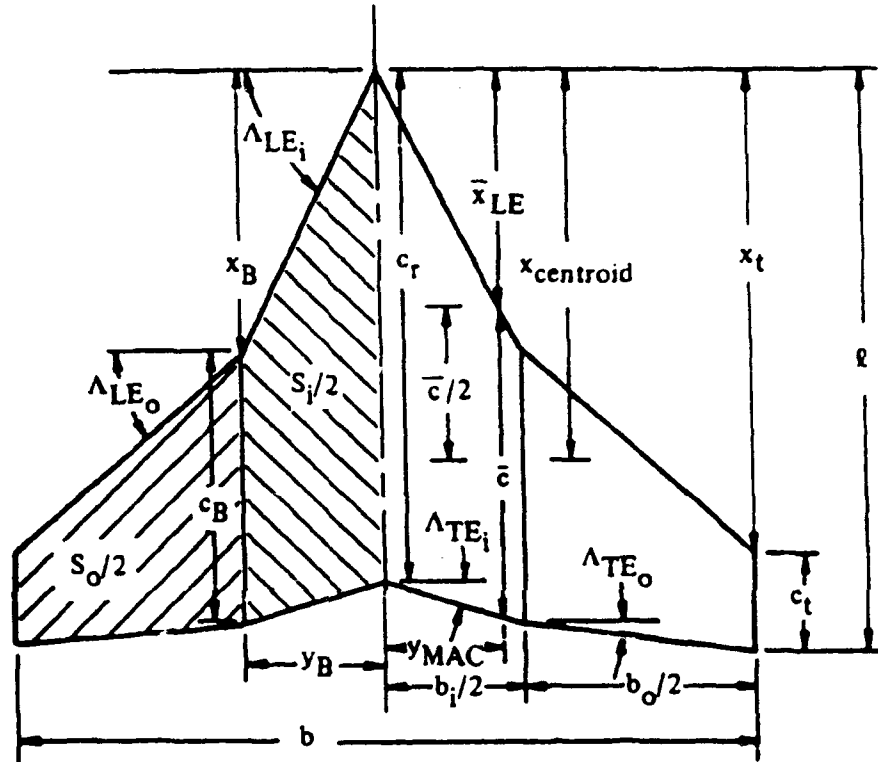


Figure VI-4 Double Delta and Cranked Wing Planform

$$AR = \frac{b^2}{S} = \frac{2b}{C_r[(1 - \lambda)\eta_B + \lambda_i + \lambda]}$$

$$S = S_i + S_o = \frac{b^2}{AR} = (b/2)C_r[(1 - \lambda)\eta_B + \lambda_i + \lambda]$$

$$\bar{c} = \frac{\bar{c}_i S_i + \bar{c}_o S_o}{S_i + S_o}$$

$$\bar{x} = \bar{x}_{LE} + \bar{c}/2$$

$$\bar{y} = \frac{\bar{y}_i S_i + [y_B + \bar{y}_o] S_o}{S_i + S_o}$$

$$\bar{x}_{LE} = \frac{[\bar{y}_i \tan \Lambda_{LE_i}] S_i + [y_B \tan \Lambda_{LE_i} + \bar{y}_o \tan \Lambda_{LE_o}] S_o}{S_i + S_o}$$

$$\bar{\eta} = \frac{\bar{y}}{b/2} = \frac{b_i \bar{\eta}_i S_i + [b_i + b_o \bar{\eta}_o] S_o}{b[S_i + S_o]}$$

$$\eta_B = \frac{b_i}{b} = \left[ \frac{1}{1-\lambda} \right] \left[ \frac{2S}{bC_r} - \lambda_i - \lambda \right]$$

$$\zeta_B = \frac{1}{\frac{b_o \tan \Lambda_{LE_o}}{1 + \frac{b_i \tan \Lambda_{LE_i}}{b_o \tan \Lambda_{LE_o}}}}$$

$$\lambda = C_t/C_r = \lambda_i \lambda_o$$

$$\lambda_i = c_B/C_r$$

$$\lambda_o = C_t/c_B$$

### Planform Example

This is an example using the equations for a cranked wing to determine the MAC,  $c/4$ , and other items of interest.

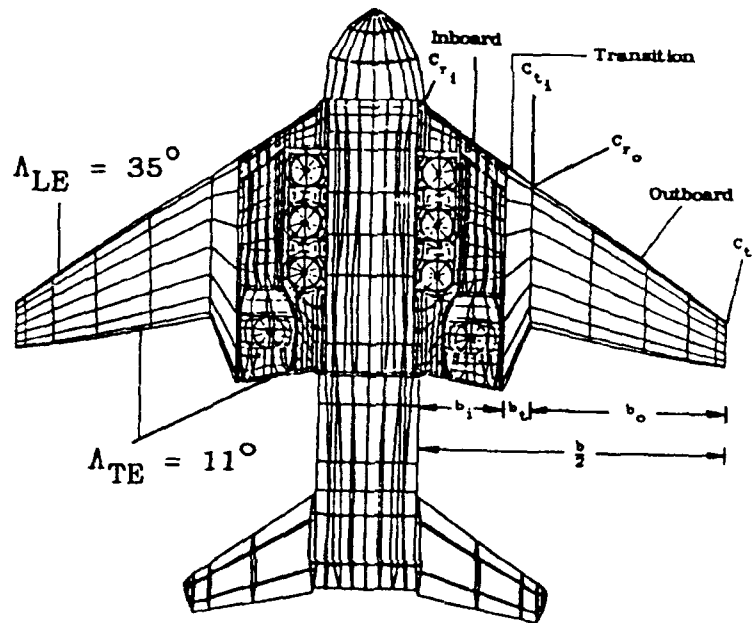


Figure VI-5 Planform Example

#### Inboard Section (i)

$$C_{r_i} = 50.45 \text{ in}$$

$$C_{t_i} = 36.51 \text{ in}$$

$$\lambda_i = C_{t_i} / C_{r_i} = 36.51 / 50.45 = 0.724$$

$$b/2 = 26.96 \text{ in}$$

$$S_i/2 = \frac{(50.45 + 36.51)}{2} (26.96) = 1172.2 \text{ in}^2 = 8.14 \text{ ft}^2$$

$$\bar{c}_i = \frac{2}{3} (50.45)(1.30) = 43.86 \text{ in}$$

$$\bar{y}_i = \left[ \frac{b_i}{2} \right] \left[ \frac{1}{3} \right] \left[ \frac{1 + 2\lambda_i}{1 + \lambda_i} \right] = (26.96)(0.473) = 12.76 \text{ in}$$

$$\begin{aligned} \bar{x}_i &= \left[ \frac{b_i}{2} \right] \left[ \frac{1}{3} \right] \left[ \frac{1 + 2\lambda_i}{1 + \lambda_i} \right] \tan \Lambda_{LE} = (12.76) \tan 35 \\ &= 8.93 \text{ in (from LE of } C_{r_i}) \end{aligned}$$

#### Outboard Section (o)

$$C_{r_o} = 24.58 \text{ in}$$

$$C_{t_o} = 7.51 \text{ in}$$

$$\lambda_o = 7.51 / 24.58 = 0.306$$

$$b_o/2 = 33.15 \text{ in}$$

$$S_o = \frac{24.58 + 7.51}{2} (33.15) = 531.89 \text{ in}^2 = 3.69 \text{ ft}^2$$

$$\bar{c}_o = \frac{2}{3} (24.58)(1.072) = 17.56 \text{ in}$$

#### Measured from LE of } C\_{r\_o}

$$\bar{y}_o = \left[ \frac{b_o}{2} \right] \left[ \frac{1}{3} \right] \left[ \frac{1 + 2\lambda_o}{1 + \lambda_o} \right] = (33.15)(.4114) = 13.64 \text{ in}$$

$$\bar{x}_o = \bar{y}_o \tan 35 = (13.64)(0.7) = 9.55 \text{ in}$$

#### Measured from LE of } C\_{r\_i}

$$\bar{y}_o = 13.64 + 26.96 = 40.6 \text{ in}$$

$$\bar{x}_o = \bar{y}_o \tan 35 = (40.6)(0.7) = 28.42 \text{ in}$$

#### Total Wing

$$S = 8.14 + 3.69 = 11.83 \text{ ft}^2$$

$$S_i/S = 0.688 \quad S_o/S = 0.312$$

$$\bar{y} = 12.76(0.688) + 40.6(0.312) = 21.45 \text{ in (1.79 ft)}$$

$$\bar{c} = 43.86(0.688) + 17.56(0.312) = 35.65 \text{ in (2.97 ft)}$$

### Total Aircraft Aerodynamic Center Location

#### Inboard Section

$$\bar{x}_{\bar{c}/4} = \bar{x}_i + \bar{c}_{c/4} = 8.93 + (43.86)/4 = 19.9 \text{ in}$$

$$\bar{y}_{\bar{c}/4} = 12.76 \text{ in (measured from } C_{r_i} \text{)}$$

#### Outboard Section

$$\begin{aligned} \bar{x}_{\bar{c}/4} &= \bar{x}_o + b_i \tan \Lambda_{LE} + \bar{c}_o \\ &= 26.96 \tan 35 + 9.55 + (17.56)/4 = 32.82 \text{ in} \end{aligned}$$

$$\bar{y}_{\bar{c}/4} = 13.64 + 26.96 = 40.6 \text{ in}$$

### Total Aircraft

#### Measured from LE of } C\_{r\_i}

$$\bar{x}_{\bar{c}/4} = 19.9(0.688) + 32.82(0.312) = 23.93 \text{ in}$$

#### Measured from } C\_{r\_i}

$$\bar{y}_{\bar{c}/4} = 21.446 \text{ in}$$

## NOTES

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## NOTES

[illegible]

VT-16

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## VII DRAG

## VII Drag

### List of Symbols

AR	Aspect Ratio
$C_D$	Total Drag Coefficient
$C_{D_0}$	Zero Lift Drag Coefficient (parasite drag)
$C_{D_{min}}$	Minimum Drag Coefficient
$\Delta C_D$	Minimum Drag Coefficient change due to camber
$C_D^*$	Drag Coefficient at $(L/D)_{max}$
$C_L$	Total Lift Coefficient
$C_L'$	Lift Coefficient at minimum drag
$C_{L_{pb}}$	Lift Coefficient at the Polar Break
$C_L^*$	Lift Coefficient at $(L/D)_{max}$
e	Span efficiency of actual aircraft drag polar
$e_p$	Span efficiency of a parabolic drag polar
g	Gravitational constant
K	Induced Drag factor
$(L/D)_{max}$	Maximum lift-to-drag ratio
Rn	Reynolds number
$V_\infty$	Freestream velocity

## Drag (ref. 13)

Drag is a part of a resultant aerodynamic force produced by the tangential (skin friction) and normal (pressure) forces acting along the vehicle's surface due to the relative fluid motion. This resultant force is resolved into the lift and drag components in the vehicle's plane of symmetry. Drag, the component of the total force that opposes motion in the equilibrium flight path direction, approximately follows a parabolic variation with lift and the angle of attack. Total drag can be resolved into various components (figure VII-1).

### Subsonic Drag

The aircraft drag, when taken below the divergent Mach number, is traditionally decomposed into lift-induced and minimum drag.

### Minimum Drag

Minimum drag can be divided into profile (skin friction, pressure, and base drag) and interference drag (figure VII-1). In general, about two-thirds of subsonic minimum drag may be attributed to profile (skin friction) drag (figure VII-2).

### Profile Drag/Skin Friction Drag (ref. 7)

The skin friction drag is due to the momentum transfer between the fluid particles adjacent to the vehicle surface and the vehicle. It (in incompressible flow) can be established for a laminar and turbulent boundary layer. This drag is based upon the "wetted" surface area of a flat plate, on ONE surface.

#### Laminar

$$C_{D_{lam}} = D_{lam} / (q S_{wet}) = 1.328 / (Rn)^{0.5}$$
$$1 \times 10^3 > Rn < 1 \times 10^6$$

### Turbulent

$$C_{D_{\text{turb}}} = D_{\text{turb}} / (q S_{\text{wet}}) = 0.455 / (\log Rn)^{2.58}$$
$$1 \times 10^6 > Rn < 1 \times 10^9$$

### Turbulent

$$C_{D_{\text{cone}}} = \frac{2}{\sqrt{3}} * C_{D_{\text{flat plate}}}$$

### Pressure Drag (Form Drag)

Pressure Drag at subsonic speeds is due to boundary layer displacement effects and separation effects on aft-facing slopes.

### Interference Drag

This drag is the result of mutual interaction of the flow fields developed by the major configuration components (ie...wing-body).

### Miscellaneous Drag

Miscellaneous drag is caused by aircraft protuberances and surface irregularities (ie... gaps, fasteners).

### Drag Due to Lift

Drag due to lift is almost entirely the result of the lift-produced circulation as well as the vortex shedding from the wing tip.

### Zero Lift Drag

Zero lift drag,  $C_{D_0}$ , is the drag at  $C_L = 0$  and can be easily found on a drag polar curve. The primary contributor to this drag is from skin friction. From figure X-6, the effect of wing sweep and Mach number on zero lift drag ( $C_{D_0}$ ) can be seen.



### Base Drag

Base drag contribution to minimum drag is caused by a rapid expansion of the flow into a base region which causes significantly reduced pressures to act upon a finite base (area).

### Internal Duct Drag

This drag is associated with a momentum loss in a flow through nacelle. This drag is measured by having a nozzle pitot static pressure rake (assume inlet conditions are at freestream conditions) measuring the momentum loss within the nacelle (duct). Once a  $\Delta C_{D_{duct}}$  is obtained, subtract it out of the total aircraft  $C_D$ .

### Wave Drag

Wave drag is primarily due to the lack of pressure recovery on the surface due to a total pressure loss through the shock wave.

### Drag Polar (Subsonic)

A drag polar can be represented by two curve types,  $C_L$  vs  $C_D$  or  $C_D$  vs  $C_L^2$ . Both types reveal aerodynamic parameters that are important to the performance of an aircraft configuration. These two drag polars can be seen in figures VII-2 through VII-5. Figures VII-2 & VII-3 display some of the major contributors to the overall total drag.

### ( $C_L$ vs $C_D$ ) Polar

This drag polar, as seen in figures VII-2, VII-3, VII-4, is parabolic (a quadratic function) by nature. The eccentricity of the parabola and its origin is affected mainly by wing camber, twist, and flow separation. A few performance parameters that can be established by this curve are  $C_{D_{min}}$ ,  $C_{D_o}$ ,  $(L/D)_{max}$ ,  $C_L$ ,  $C_{L_{pb}}$ . A drag polar has the characteristic equations listed below.

### Parabolic

$$C_D = C_{D_{\min}} + \frac{C_L^2}{\pi A Re_p}$$

### NON-Parabolic (camber, twist, etc.. effects)

$$C_D = C_{D_{\min}} + \frac{(C_L - C_L')^2}{\pi A Re}$$

### (C<sub>D</sub> vs C<sub>L</sub><sup>2</sup>) Polar

From figure VII-5, the slope of this line is K, the induced drag factor. And from K, Oswald's wing efficiency factor, e, can be obtained. Also, from figure VII-5's cutout, the intersection of the C<sub>D</sub> axis and the curve reveals C<sub>D<sub>0</sub></sub>.

### Polar Break (ref. 14)

(Subsonic)

When the wing leading edge suction has lost its force (separation has occurred), the polar shape departs drastically from the typical parabolic shape. The drag polar is said to "break" at this point (figure VII-2). The break point is sensitive to Mach number, Reynolds number, leading edge radius, and wing geometry.

### Camber Effects (ref. 14)

Camber (and twist) essentially shifts the drag polar. The effect of camber (and twist) can be seen in  $\Delta C_{D_{\min}}$  resulting in a higher C<sub>D<sub>0</sub></sub> and C<sub>D<sub>min</sub></sub>. The parabolic extent of the polar is increased and its shape is improved through the use of camber (and twist).

Drag and Performance Equations (ref. 14)

$$C_D = C_{D_{\min}} + K(C_L - C_L')^2$$
$$C_{D_{\min}} = C_{D_{\min \text{ parabolic}}} + \Delta C_{D_{\min}}$$

$$C_L' = (1 - \frac{e}{e_p}) * C_{L_{pb}}$$

$$\Delta C_{D_{\min}} = \left[ \frac{e_p - e}{\pi A R e_p^2} \right] * C_{L_{pb}}^2 = \frac{C_L'^2}{\pi A R (e_p - e)}$$

$$e = \frac{(C_L - C_L')^2}{\pi A R (C_D - C_{D_o})}$$

NON-Parabolic Polar

$$C_L^* = \sqrt{\frac{C_{D_o}}{K}}$$

$$C_D^* = 2(C_{D_o} - C_L' \sqrt{K C_{D_o}})$$

$$(L/D)_{\max} = \frac{1}{2(C_{D_o} - C_L' \sqrt{K C_{D_o}})}$$

Parabolic Polar

$$C_L^* = \sqrt{\frac{C_{D_o}}{K}}$$

$$(L/D)_{\max} = \frac{1}{2\sqrt{K C_{D_o}}}$$

$$C_D^* = 2C_{D_o}$$

### Analytically Determined Drag Polar (ref. 15)

Aerodynamic wind tunnel data (drag) is never exactly parabolic. But, the data can be approximated by a parabolic relationship. The approach is as follows.

$$C_D = C_{D_{\min}} + K(C_L - C_L')^2 \quad (1)$$

$$C_{D_o} = C_{D_{\min}} + K(C_L')^2$$

where  $C_{D_{\min}}$ ,  $K$  and  $C_L'$  are unknown; expanding equation 1

$$C_D = C_{D_o} - 2KC_L' C_L + KC_L^2$$

$$C_D = a + bC_L + cC_L^2$$

### Method of Determining a Drag Polar

Assume "N" matched points of [CL(i), CD(i)]

$aN$	$+ b \sum_{i=1}^N C_L(i)$	$+ c \sum_{i=1}^N C_L(i)^2$	$= \sum_{i=1}^N C_D(i)$
$a \sum_{i=1}^N C_L(i)$	$+ b \sum_{i=1}^N C_L(i)^2$	$+ c \sum_{i=1}^N C_L(i)^3$	$= \sum_{i=1}^N C_L(i) * C_D(i)$
$a \sum_{i=1}^N C_L(i)^2$	$+ b \sum_{i=1}^N C_L(i)^3$	$+ c \sum_{i=1}^N C_L(i)^4$	$= \sum_{i=1}^N C_L(i) * C_D(i)^2$

Once  $\sum_{i=1}^N C_L(i)$ 's &  $\sum_{i=1}^N C_D(i)$ 's are determined, use a Gaussian Elimination technique to solve for the coefficients  $a$ ,  $b$ ,  $c$  where  $a = C_{D_o}$ ,  $b = -2KC_L'$ ,  $c = K$

Example

$$N = 5$$

i	$C_L$	$C_D$
1	0.0	0.0190
2	0.1	0.0202
3	0.2	0.0234
4	0.3	0.0313
5	0.4	0.0456

Resulting augmented matrix

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & .3 & .1 \\ .3 & .1 & .0354 \end{bmatrix} \begin{bmatrix} 0.1395 \\ 0.03433 \\ 0.11251 \end{bmatrix}$$

$$a = 0.01945$$

$$b = -0.02398$$

$$c = 0.2207$$

$$\begin{array}{lcl} C_{D_o} & = & 0.0194 \\ K & = & 0.2207 \\ C_L' & = & 0.0543 \end{array}$$

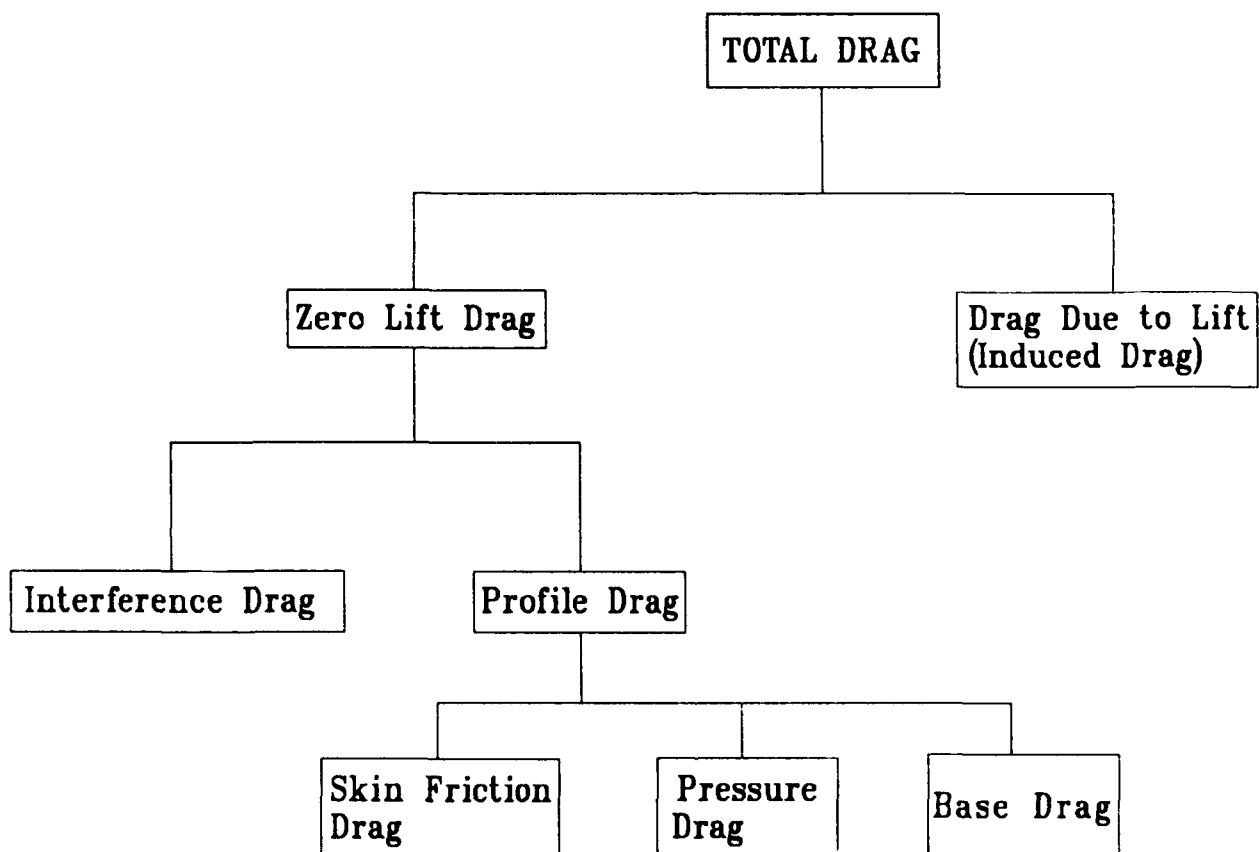


Figure VII-1 Drag Tree

## Subsonic

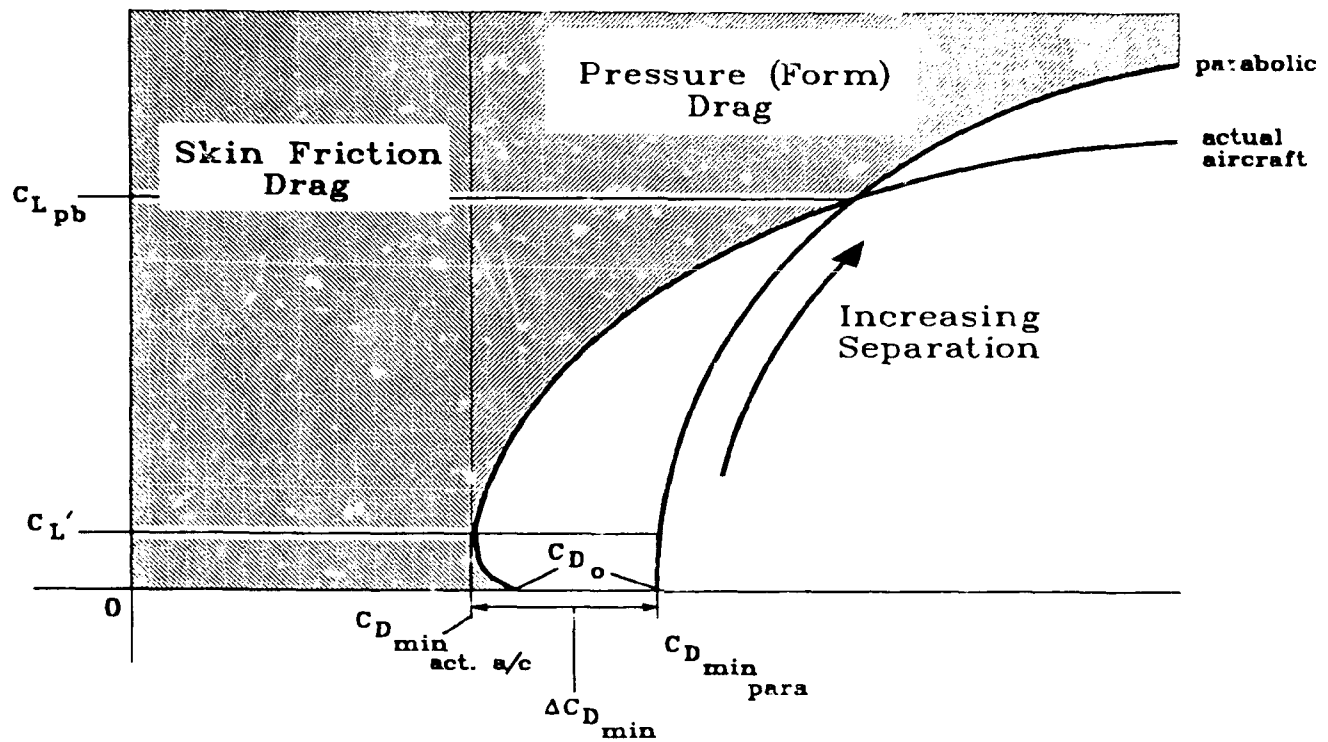


Figure VII-2 ( $C_L$  vs  $C_D$ ) Drag Polar

## Supersonic

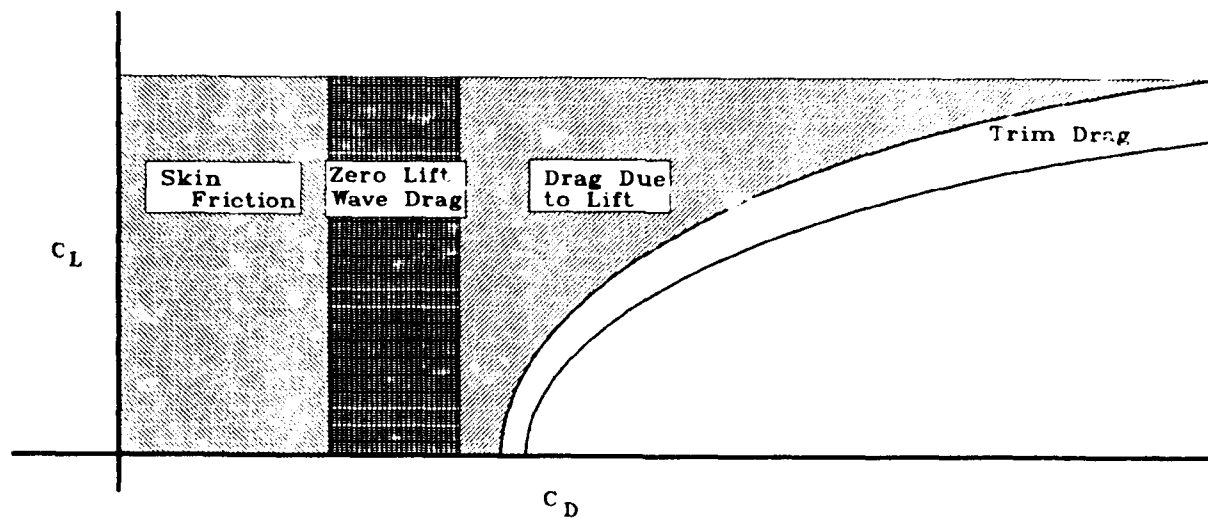


Figure VII-3 ( $C_L$  vs  $C_D$ ) Drag Polar

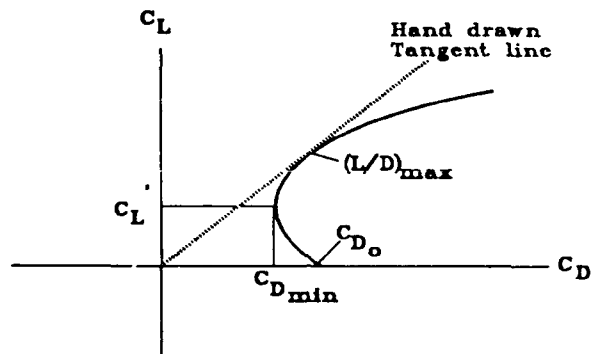


Figure VII-4 ( $C_L$  vs  $C_D$ ) Subsonic Drag Polar

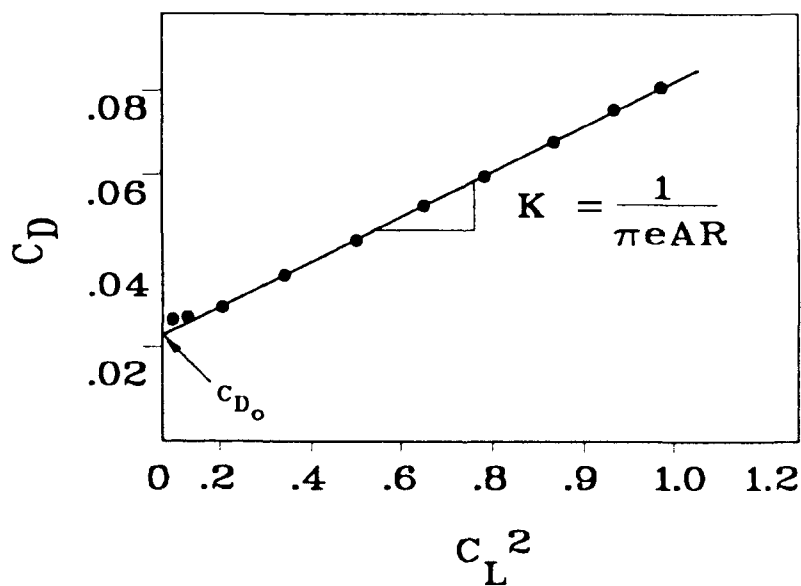


Figure VII-5 ( $C_D$  vs  $C_L^2$ ) Drag Polar



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## NOTES

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## NOTES

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## NOTES

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## VIII EXPERIMENTAL TESTING AND INTERPRETATION

## VIII Experimental Testing and Interpretation

(Ref. 8)

For each wind tunnel test accomplished, the items of interest for the engineer will vary. Below is a list of items and facts that are usually of interest to the engineer. Reference 8 is an excellent aid for finding those items and for subsonic wind tunnel testing. This handbook and reference 8 should be the two reference materials that accompany the testing engineer to the wind tunnel site.

### Aircraft (wing-body-tail)

$C_{L_{\max}}$	$C_{L_{\alpha}}$	$C_{M_{\alpha}}$	$C_{M_o}$
$C_D$	$C_{D_{\min}}$	$C_{D_o}$	$\alpha_o$
$(L/D)_{\max}$	$C_{L_o}$	$C_{M_{ac}}$	$C_{n_{\beta}}$
$C_{l_{\beta}}$	$C_{Y_{\beta}}$		

### Flaps

The purpose of flaps is to reduce the wing area through increasing  $C_{L_{\max}}$  and thus reduce the parasite (skin friction) drag in cruise. Flap systems usually generate large negative pitching moments and require a large horizontal tail to develop adequate down loads for trim, which reduce the total (wing-body-tail)  $C_{L_{\max}}$ .

### Trailing Edge Flaps (figure VII-9)

- Reduce  $\alpha$  for  $\alpha_o$  ( $\alpha$  zero lift)
- Increase  $C_{L_{\max}}$
- Increases  $\alpha$  for stall

### Leading Edge Flaps

- Extend lift curve to increase  $\alpha_{\text{stall}}$
- Extend  $C_{L_{\max}}$

### Lift Curve (Flaps Up)

- Used to determine flap-up stalling velocity
- $C_{L_{max}}$  range from 0.6 to 1.7 (unpowered)
- Wing  $C_{L_{max}}$  runs 85% to 90% of airfoil values (with no high lift devices)
- $C_{L_{max}}$  usually increases with Reynolds number (Rn)
- L.E. slats are insensitive to Rn, but slats reduce the effect of Rn on  $C_{L_{max}}$
- $C_{L_{max}}$  for flaps retracted is less than  $C_{L_{max}}$  flaps extended
- Substantial variations of  $C_M$  often occur at  $C_{L_{max}}$  or ( $\alpha_{stall}$ )
- Model should be as close to trim as possible
- Nacelles usually reduce  $C_{L_{max}}$
- To be usable,  $C_{L_{max}}$  must be for trimmed flight
- $\alpha$  at  $0.9 C_{L_{max}}$  is of interest for landing gear length consideration
- Stall  $\alpha$  should be taken in very small steps so that its shape (non-linear portion of  $C_L$  curve) and  $C_{L_{max}}$  and  $\alpha_{stall}$  can be determined accurately
- To increase the span of the wing affected by the flaps (increasing  $C_{L_{max}}$ ) the ailerons can be drooped

### Lift Curve (Flaps Down)

- $C_{L_{max}}$  flaps down range generally from 1.2 to 3.5 (unpowered)
- Should have approximately same slope ( $C_{L_{\alpha}}$ ) as flaps up
- Should have same location for the aerodynamic center as flaps up
- Used to find  $\Delta C_{L_{max}}$  due to flap deflection



- There is usually little need to take the flap- down lift curve as low as  $\alpha_o$

#### Drag Curve (Flaps Up and Down)

- Near  $C_{D_{min}}$ , step  $\alpha$  in one degree increments
- $C_{D_{min}}$  (clean fighter)  $\approx 0.0120$  (120 counts of drag)
- The shape of the drag curve is important for climb and cruise with small changes in drag due to lift being desired during these portions of the missions.
- The value of  $C_D$  at  $C_{L_{max}}$  is needed for takeoff and landing calculations.

#### Pitching Moment

- $C_{m_\alpha}$  must be negative for longitudinal stability.
- Generally, the pitching moment curves are used to determine if the aircraft has static stability through the desired C.G. range at all flight conditions (trimmed flight).

#### Elevator or Stabilizer Power Curve

- Plotting  $\Delta C_M$  versus  $\delta_e$ , elevator deflection, (stab. incidence) is made at several  $C_L$ 's. This plot indicates the amount of elevator or stabilizer deflection is needed to produce a certain moment coefficient.
- Stabilizer (elevator) effectiveness ( $dC_M/d\delta_e$ ) is obtained by holding  $\alpha_{wing}$  constant and varying the tail incidence.
- Plot  $C_{M_{cg}}$  versus  $C_L$  for several elevator angles. The intersections of the curves with the axis indicate trim conditions (figure VII-7). By holding  $\alpha_{wing}$  constant and varying stabilizer incidence, the pitching moment about the tail is  $M_t = -l_t q_t S_t C_{L_t}$ . When  $l_t$ ,  $S_t$  are known, then  $C_{L_t}$  can be found. Use  $q_t = 0.85q_\infty$ . From  $C_{L_t}$  and known stabilizer angles,

the slope of the tail lift curve ( $dC_{L_t}/d\alpha_t$ ) can be established.

- Elevator power must be sufficient to balance ( $C_{M_{cg}} = 0.0$ ) the airplane at maximum lift. The critical condition is gear and landing flaps down and in ground effects.
- On low aspect ratio configurations, with short tails, tail effectiveness varies with  $\alpha$  as the local dynamic pressure changes.
- On swept wing configurations, attention must be paid to pitch-up (reversal of  $C_M$  curve) in the  $C_L$  vs  $C_M$  curve as it can limit usable  $C_L$ .

### Aileron Power Curves

- Aileron criteria are usually determined at  $\phi = 0^\circ$  and plots of  $C_l$  vs  $C_n$ ,  $C_l$  vs  $\delta_a$ , and  $C_l$  vs  $\delta_a$  are used.
- Good qualities of ailerons are high rolling moments and low hinge moments.
- Maximum roll rate and maximum helix angles are determined from  $C_{l_{max}}$ .  
Generally,  $C_{l_{max}}$  of 0.03 is adequate for one aileron.
- When yaw ( $\phi$ ) equals 0, and there is a slight rolling or yawing moment or side force when controls are neutral, be sure to subtract them out before reporting the data. This delta can be attributed to asymmetrical tunnel flow or model asymmetry.
- $\partial C_l / \partial \phi = 0.0002$  is equivalent to  $1^\circ$  of effective dihedral.

If there is enough time and money (which usually there is not), do the aileron effectiveness test with tail off (ref. 8). The first reason to do this is when the ailerons are deflected in flight, the aircraft normally rolls and the inboard aileron trailing vortices are swept away from the horizontal tail by the helix angle. In the wind tunnel, these vortices stream back close to the horizontal tail and induce a load on the tail that does not occur in flight. Secondly, it saves effort in data reduction since tunnel wall effects on a horizontal tail is then non-existent.

### Rudder Power Curves

- Rudder power on high-performance multi-engine aircraft must possess sufficient directional stability to prevent excessive yaw angles ( $\varphi$ ).
- Rudder power must be able to be balanced directionally at best climb speeds with asymmetric power (engine out).
- Rudder equilibrium is a plot of rudder deflection,  $\delta_r$ , versus yaw angle  $\varphi$ .
- When doing a rudder study, deflect the rudder and yaw the model only in one direction. You are allowed to do this due to model symmetry.
- Rudder Equilibrium:  $\partial\varphi / \partial\delta_r$  ranges from -1.2 to -0.5 (maneuverable to stable)
- Rudder Power:  $\partial C_n / \partial\delta_r = -0.001$  is reasonable.

### Determine Center of Pressure Shift (C.P.)

To determine the C.P. shift the derivative  $\partial C_M / \partial \text{Mach}$  is used. The main factor that contributes to this derivative is the backward shift of the wing center of pressure (C.P.) which occurs in the transonic range. On two-dimensional symmetrical wings, for example, the C.P. moves from approximately 0.25c to approximately 0.5c as the Mach number increases from subsonic to supersonic values.

### C.G. Shift

Moving the C.G. forward reduces  $\alpha_{\text{trim}}$  or  $C_L$  resulting in an increase in trim speed.

### Lift Curve Slope $C_{L\alpha}$

- $C_{L\alpha}$  ranges approximately  
0.110 per degree for thin airfoils  
0.115 per degree for thick airfoils }  $R_n > 10^6$
- As a rule of thumb;  $C_{L\alpha_t}$  is usually < 90% of  $C_{L\alpha}$ .
- $C_{L\alpha}$  makes an important contribution to the dampening of the longitudinal short period mode.

## Determining Aircraft Parameters from Wind Tunnel Data (ref. 29)

Wing-Body wind tunnel data:

$\alpha$	-1.5	5.0
$C_L$	0.0	0.52

$\alpha$  = geometric angle-of-attack

$\alpha$	1.0	7.88
$C_M$	-0.01	0.05

$C_M = C_{Mcg_{wb}}$

### Static Margin

Lift curve slope

$$a_{wb} \equiv \frac{\partial C_L}{\partial \alpha} = \frac{0.52 - 0}{5 - (-1.5)} = 0.08 \text{ per degree}$$

$$C_{Mcg_{wb}} = C_{Mac_{wb}} + a_{wb} \alpha_{wb} (h - h_{ac_{wb}}) \quad (1)$$

$\alpha_{wb}$  = absolute angle-of-attack

at  $\alpha = 1.0^\circ$

$$-0.01 = C_{Mac_{wb}} + 0.08 (1 + 1.5)(h - h_{ac_{wb}}) \quad (2)$$

at  $\alpha = 7.88^\circ$

$$0.05 = C_{Mac_{wb}} + 0.08(7.88 + 1.5)(h - h_{ac_{wb}}) \quad (3)$$

Equations 2 & 3 can be solved simultaneously. Subtracting equation 3 from equation 2...

$$-0.06 = 0 - 0.55(h - h_{ac_{wb}})$$

$$(h - h_{ac_{wb}}) = \frac{-0.06}{-0.55}$$

$$(h - h_{ac_{wb}}) = 0.11 \quad (\text{static margin})$$

### Aerodynamic Center Location

$$h = 0.35\bar{c}$$

$$(h - h_{ac_{wb}}) = 0.11$$

$$h_{ac_{wb}} = 0.35 - 0.11$$

$$h_{ac_{wb}} = 0.24 \quad (\text{a.c. location \% } \bar{c})$$

### Aerodynamic Center Pitching Moment

Using equation 1 ...

$$-0.01 = C_{Mac_{wb}} + 0.08(1 - 1.5)(0.11)$$

$$C_{Mac_{wb}} = -0.032$$

### Center-of-Gravity Pitching Moment

For a given wing-body combination, the aerodynamic center lies 0.05 (5%) of a chord length ahead of the c.g.. The moment coefficient about the aerodynamic center is -0.016. If the lift coefficient is 0.45, calculate the moment coefficient about the c.g..

$$C_{Mc_{g_{wb}}} = C_{Mac_{wb}} + C_{L_{wb}}(h - h_{ac_{wb}}) \quad (4)$$

$$(h - h_{ac_{wb}}) = 0.05 \quad C_{L_{wb}} = 0.45 \quad C_{Mac_{wb}} = -0.016$$

$$C_{Mc_{g_{wb}}} = -0.016 + 0.45(0.05)$$

$$C_{M_{cg_{wb}}} = 0.0065$$

### Wing-Body-Tail

Consider the wing-body data above, the area and the M.A.C. of the wing are 1.076 ft<sup>2</sup> and 0.328 ft respectively. The distance from the airplane c.g. to the tail a.c. is 0.557 ft, the tail area is 0.215 ft<sup>2</sup>, the tail setting angle is 2.7°, the tail lift-slope is 0.1 per degree, and from experimental measurement,  $\epsilon_o = 0.0$  and  $\partial\epsilon/\partial\alpha = -.35$ . If  $\alpha = 7.88^\circ$ , calculate  $C_{M_{cg}}$ .

From the information above ..

$$C_{M_{cg}} = C_{M_{ac_{wb}}} + a\alpha_a \left[ (h - h_{ac_{wb}}) - V_H \frac{a_t}{a} \left[ 1 - \frac{\partial\epsilon}{\partial\alpha} \right] \right] + V_H a_t (i_t + \epsilon_o) \quad (5)$$

$$C_{M_{ac_{wb}}} = -0.032$$

$$\alpha_a = 7.88 + 1.5 = 9.38^\circ$$

$$a_{wb} = 0.08$$

$$(h - h_{ac_{wb}}) = 0.11$$

$$V_H = \frac{l_t S_t}{\bar{c} S_w} = \frac{(.557)(.215)}{(.328)(1.076)} = 0.34$$

$$a_t = 0.1/\text{deg}$$

$$\partial\epsilon/\partial\alpha = 0.35$$

$$i_t = 2.7^\circ$$

$$\epsilon_o = 0.0$$

Using Equation 5

$$C_{M_{cg}} = -0.032 + (.08)(9.38) \left[ 0.11 - .34 \left[ \frac{.1}{.08} \right] (1 - .35) \right] + .34(.1)(2.7 + 0.0)$$

$$C_{M_{cg}} = -0.032 - 0.125 + 0.092$$

$$C_{M_{cg}} = \boxed{-0.065}$$

### Longitudinal Static Stability

Does the aircraft (wing-body-tail) above have longitudinal static stability and balance ?

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = a \left[ (h - h_{ac_{wb}}) - V_H \left[ \frac{a_t}{a} \right] \left[ 1 - \frac{\partial \epsilon}{\partial \alpha} \right] \right] \quad (6)$$

$$a_{wb} = 0.08$$

$$(h - h_{ac_{wb}}) = 0.11$$

$$V_H = 0.34$$

$$a_t = 0.1 \text{ per degree}$$

$$\partial \epsilon / \partial \alpha = 0.35$$

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = 0.08 [(0.11) - 0.34] \left[ \frac{0.1}{0.08} \right] (1 - 0.35)$$

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = \boxed{-0.0133}$$

Since the slope is negative, thus the aircraft is statically stable.

### Longitudinal Balance

$$C_{M_o} = C_{M_{ac_{wb}}} + V_H a_t (i_t + \epsilon_o) \quad (7)$$

$$C_{M_{ac_{wb}}} = -0.032 \quad i_t = 2.7^\circ$$

$$C_{M_o} = -0.032 + (0.34)(0.1)(2.7)$$

$$C_{M_o} = \boxed{0.06}$$

### Trim angle-of-attack

Remember that  $\frac{\partial C_{M_{cg}}}{\partial \alpha_{wb}}$  is the slope of a straight line. Therefore by setting  $C_{M_{cg}}$  to zero,

and writing the equation of a straight line

$$y = mx + b \quad (8)$$

$$\begin{aligned} y &= C_{M_{cg}} & m &= \frac{\partial C_{M_{cg}}}{\partial \alpha_a} \\ x &= \alpha_{trim} & b &= C_{M_o} \end{aligned}$$

$\alpha_{trim}$  can be found. From the above example...

$$0.0 = 0.06 - 0.0133\alpha_{trim}$$

$$\boxed{\alpha_{trim} = 4.5^\circ}$$

Clearly, this angle-of-attack falls within the reasonable flight range. Therefore, the aircraft is longitudinally balanced and statically stable.

#### Finding Trimmed Flight Parameters (ref. 16)

(Unpowered, No thrust components)

Trimmed flight parameters can be easily found from a  $C_M$  vs  $C_L$  and  $C_L$  vs  $\alpha$  plot. One of the model test parameters an engineer should always consider testing is the elevator. From this data, trim conditions can be found. When testing the elevator for its effectiveness, the angle-of-attack for the model configuration should be held constant while only a change of the elevator deflection angle ( $\delta_e$ ) is accomplished. Holding the configuration at a constant angle-of-attack is not necessary, but it helps when manipulating the data for presentation. Make as many runs as necessary at different  $\delta_e$ 's while the horizontal tail incident angle is held constant. The resultant plot can be seen in figure VII-7. On the  $C_M$  vs  $C_L$  plot, where each  $\delta_e$  curve crosses  $C_M = 0$ , that point is considered a trim point and the corresponding  $C_L$  is the trim  $C_L$ . To find  $\alpha_{trim}$ , a line (this line equates to a constant  $C_L$ ) is drawn from the  $C_M$  vs  $C_L$  plot to the corresponding  $\delta_e$  on the  $C_L$  vs  $\alpha$  plot. That intersection on the  $C_L$  vs  $\alpha$  plot  $\alpha_{trim}$  can be found. If a series of trim points are found on the  $C_L$  vs  $\alpha$  plot a  $C_{L_{\alpha_{trim}}}$  can also be found.



### Determining any C.G. Location

(unpowered, gliding flight)

To find the trim envelope (where  $C_M = 0$ ) for any c.g. location, a line of any slope is drawn by rotating that line from  $C_M = C_L = 0$  on the  $C_M$  vs  $C_L$  curve to any position on the  $C_M$  vs  $C_L$  curve. The slope of the newly drawn line is equal to the c.g. shift (refs. 16 and 8).

$$\left. \frac{\partial C_M}{\partial C_L} \right|_{\text{new c.g. position}} = \left. \frac{\partial C_M}{\partial C_L} \right|_{\text{old c.g. position}} - \Delta$$

where

$$\Delta = [\text{old c.g. position (\%MAC)} - \text{new c.g. position (\%MAC)}]$$

or

$$\Delta = [(x_{cg} - x_{ac})_{\text{old}} - (x_{cg} - x_{ac})_{\text{new}}]$$

Example:

Move c.g. from 0.35 to 0.2;

$$\left. \frac{\partial C_M}{\partial C_L} \right|_{0.35} = 0.2$$

$$\left. \frac{\partial C_M}{\partial C_L} \right|_{0.2} = 0.2 - (0.35 - 0.2)$$

$$\left. \frac{\partial C_M}{\partial C_L} \right|_{0.2} = 0.2 - 0.15$$

$$\left. \frac{\partial C_M}{\partial C_L} \right|_{0.2} = 0.05$$

It appears that to acquire this transfer a subtraction of moment arms ( $\Delta$ ) was accomplished. However, in reference 16, a thorough explanation is discussed and it shows that it is not just subtraction of moment arms but rather a subtraction of moments.

### Determining the Average Downwash Angle (ref. 8)

To determine the average downwash behind the wing test the model configuration with the

tail-off. Then test the configuration with the tail-on at different tail incident angles. The tail incident angle ( $i_t$ ) is the angle between the wing-body zero-lift line (generally the longitudinal axis) and the tail zero-lift line. If the tail is a symmetric airfoil, the tail zero lift line and the tail chord line are the same. Also, if the tail is an all movable tail, the tail incident angle would generally be the same as  $\delta_e$ . Once the tail-on and tail-off data are acquired, plot the data similar to figure VIII-1 ( $\alpha_w$  vs  $C_M$ ) at different  $i_t$ 's and then plot the tail-off data. The intersection of the tail-on curve with the tail-off curve are points where, at a given  $\alpha_w$ , the tail-on  $C_M$  equals the tail-off  $C_M$ . Also, at those intersections, the tail is at zero lift.

(For a symmetrical airfoil section)

$$\alpha_t = \alpha_w + i_t - \epsilon_w = 0$$

$\epsilon_w$  = wing downwash angle

$\alpha_t$  = Tail angle-of-attack

$$\boxed{\epsilon_w = \alpha_w + i_t}$$

Once  $\epsilon_w$  is found then the parameter  $\frac{\partial \epsilon}{\partial \alpha}$  can easily be obtained by plotting  $\epsilon_w$  vs  $\alpha$ .

Determining Induced Drag Factor (K)

and Oswald's Wing Efficiency Factor (e)

Draw a plot of  $C_D$  vs  $C_L^2$ . The slope of this line is K, the induced drag factor (see figure VII-5). The intercept is  $C_{d_0}$ .

$$\text{Since } K = \frac{1}{\pi A R e}$$

Oswald's wing efficiency factor (e) is...  $e = \frac{1}{\pi A R K}$

Base Pressure (ref. 4)

A base pressure correction is applied to remove the base pressure drag from the total drag in order to correct the force drag. Such a correction is required because of the base drag is unknown without the interference or interactions of the sting or a jet. The measured base pressure is corrected to the reference ambient static pressure which is

considered to act over the entire base of the model. Occasionally more than one static pressure is measured on the base of the model and averaged to arrive at the base pressure. This average is multiplied by the area of the base to obtain the base-pressure axial force correction. In equation form:

$$F_A = (C_{P_{ave}}) (A_B)(q_\infty)$$

Where

$F_A$  = Base pressure axial force to be added to the measured axial force

$C_{P_{ave}}$  = Average pressure coefficient on base,

$P_\infty$  = Free stream static pressure (in test section)

$q_\infty$  = Free stream dynamic pressure

$A_B$  = Base area

### Pressure Transducer Selection

To determine the appropriate differential pressure transducer, three items are needed. The first being the maximum expected pressure coefficient (CP), the second being the minimum expected CP and finally the dynamic pressure (q). If obtaining the maximum and minimum CP's are difficult, then use +0.5 and -2.0 for the first iteration (these suggested CP's are good numbers for wing pressures). Then just "plug and chug" the equation below.

$$CP = \frac{p - p_\infty}{\frac{1}{2}\rho V^2}$$

$$\Delta p = (p - p_\infty) = CP(q^*)$$

\* Differential pressure transducers are usually rated in psi ( $\frac{lbf}{in^2}$ ).

Conversion of 'q' to psf ( $\frac{lbf}{ft^2}$ ) may be required.

From the maximum or minimum CP determine the greatest magnitude of  $\Delta p$  and then acquire the appropriate pressure transducer.

Example:

$$CP1 = +0.5 \quad CP2 = -2.0 \quad q = 2.11 \text{ psi}$$

$$\Delta p1 = 1.055 \text{ psi} \quad \boxed{\Delta p2 = -4.22 \text{ psi}}$$

Based upon  $\Delta p2$ , a differential pressure transducer of 5 psi will be adequate.

### Flow Visualization

Flow visualization offers the testing engineer a unique way to observe the local flow fields. Flow visualization can be separated into two categories, surface flow visualization and off-body flow field visualization. Techniques to observe surface flow fields are tufts and oil flows. Laser light sheet, smoke, and tufts are used for off-body flow fields.

#### Surface Flow Visualization

##### Yarn

##### Tufts do affect the aerodynamic forces.

Yarn should be used when the testing engineer is not concerned with the model forces and moments. Yarn has the greatest adverse effect on lift and drag compared to other surface flow visualization techniques. For yarn tufts, use 0.75 inch length of No. 6 floss crochet yarn (any color). Have plenty of yarn available since it does not last a long time in the wind tunnel. Before applying tufts on the model, clean the model with naphtha or other solvents to remove any oils. There are two methods used to apply the yarn tufts to the model. The first method is to use a tuft board. A tuft board is a piece of scrapwood with two nails in it. The distance between the two nails is the length of the tuft (generally 0.75 to 1.0 inch). Wrap the tufts around the nails and then cut the tufts on the backside of the nail to give you the correct length that is needed. To attach the yarn to the model use cellophane tape or "super glue" and apply the yarn to the model in a symmetric pattern (0.75 inch x 0.75 inch). Ensure that at least 0.75 inch of the tuft material is available for the flow to manipulate freely. The second method in applying tufts to the model is to tape the yarn at two opposite ends of the item of interest on the model (ie... leading to the trailing edge) and glue/tape the tuft at the desired interval lengths (0.75 in) then cut the yarn.

### Fluorescent Mini-tufts (ref. 26)

Fluorescent mini-tufts allow a large number of tufts to be applied to a model surface in a manner that produces negligible interference with model forces and pressures. Comparisons of tufts-on and tufts-off from Mach 0.5 to 2.4 showed differences of two to three drag counts. Mini-tufts are an extremely fine nylon mono-filament fiber that has been treated with a fluorescent dye that renders it visible during fluorescence (ultraviolet) photography. There are two sizes of mini-tufts generally used during wind tunnel testing. Those sizes are 3 denier (0.02mm, 0.0007 inch diameter) and 15 denier (0.04mm, 0.0017 inch diameter). Free moving length (measured from the gluing point) of the mini-tufts should range from 0.5 to 0.75 inches.

### Mini-Tuft Installation

The procedures listed below have proven to be adequate for most applications. Tuft application utilizing these procedures are able to withstand many hours of testing at transonic and supersonic Mach numbers without appreciable adhesion failure.

### Surface Preparation Steps

1. Wipe model surface with a solvent to remove grease and oil. Use clean paper towels or tissues rather than shop rags. Preferred solvents are naphtha or any chlorinated hydrocarbons such as trichloroethylene or trichloroethane. Methyl ethyl ketone or Freon are less suitable because of rapid evaporation rates. Do not use alcohols.
2. If possible, lightly abrade the surface with a fine grit carborundum pad or aluminum oxide paper. Do not use silicon carbide paper on aluminum. Care should be taken not to compromise the aerodynamic smoothness requirements for the model.
3. After abrading the surface, thoroughly clean the surface with a solvent using clean tissues. Then wipe that area with another clean tissue using that tissue only once. Wiping more than once with the same tissue redeposits the contaminants. Continue to wipe the surface with solvent until the tissue shows no stains. Avoid letting the solvent evaporate completely before it is wiped.
4. Apply an Alodine solution to the surface with a swab or a brush. Allow it to remain wet on the surface for three to five minutes. Any areas that resist wetting should be scrubbed with carborundum pad wet with the Alodine solution (Alodine is a chromate conversion solution that promotes adhesion of the gluing material to the surface. It

is available from aircraft paint suppliers. In this application, add a wetting agent such as Kodak Photo-Flo<sup>®</sup> to Alodine to make it a one percent concentration.).

5. Rinse the Alodine from the surface with clean water and then dry the surface with clean tissues. Be careful to limit skin contact with the Alodine and to wash thoroughly after use.
6. Cover the cleansed area with paper to avoid recontamination until after the tuft adhesive is applied. Realize that the above procedures are involved and can be reduced in scope. Steps one through six are the correct way to prepare the model surface. However, there is a correct way of doing things and the getting it done way of doing things. Be flexible in this area.

#### Mini-Tuft Attachment Steps

1. Lay out alignment marks with a soft pencil (roughly 0.75 inch grid) so that the tufts can be applied in a symmetrical pattern.
2. Place oversized lengths of tuft material over the extreme ends of the model (i.e... L.E. to T.E. if a wing) surface by wrapping the tuft around the model and then taping it to the lower surface.
3. Be careful to use only slight tension to avoid stretching the filament.
4. Use a "super glue" type of gluing agent to apply the tufts to the surface. Apply a small drop of glue to the strand of tufting material in intervals of approximately 0.5 to 0.75 inches.
5. Be sure glue drops are dried before cutting the tuft material. Cut tufts just ahead of each adhesive drop with a new razor blade.

Experience has shown that the visual appearance of these tuft results are greatly enhanced by carefully keeping the tuft spacing uniform. If the tuft pattern is asymmetrical, it is still possible to describe the flow, but it is difficult to interpret. To illuminate the mini-tufts, an ultraviolet light source is required. The ultraviolet illumination excites the fluorescent material to radiate in the visible spectrum (i.e., 400-600 nanometers). Video tape or black and white still photographs are the normal surface flow visualization data medium. It also helps if a fluorescent felt tip marker is used to hi-lite/outline the model. This will allow the model lines to be seen in the photograph or video tape.

## Oil Flow

Oil flow visualization shows flow separation lines, vortex reattachment lines, shock lines, and complete boundary layer activity on the surface. The oil flow runs should be held for the last runs of the test since the oil will clog static pressure taps and is generally a messy procedure. The model is painted a flat color with the oil a color that will contrast the model color (generally a black model and white oil). The oil viscosity is an important parameter to be considered. If the oil is too viscous it will not display the true surface flow. If the oil is not viscous enough, it will run off the model and not display the flow on the surface. There are several formulas that can be used for the oil. The formulas will differ from tunnel-to-tunnel and from test engineer-to-test engineer based on his or her experience. One formula that is used and works well is one teaspoon of titanium dioxide (a white color), one teaspoon of STP Oil Treatment<sup>®</sup>, and five drops of Oleic acid. Mix this amalgamation thoroughly and apply it on the model using a syringe with a 18 or 20 gauge needle. When applying the oil to the model it should have a logical and somewhat symmetrical pattern.

## Off Body Flow Visualization

### Laser Light Sheet (Vapor Screen)

The vapor screen technique is a simple, yet effective, flow visualization tool to study the off-body flows about aerodynamic shapes at subsonic, transonic, and supersonic speeds. In recent years, this technique has frequently been employed in wind tunnel experiments to improve the understanding and control of the vortices shed from slender bodies of missiles, fuselage forebodies, and wings of fighter aircraft at high angles-of-attack. The technique features the injection of sufficient water into the tunnel circuit to create a condensation in the test section. An intense sheet of light is generated, usually with a laser (18-watt Argon-ion laser is sufficient), that can be oriented in any selected plane relative to the test model. The light is scattered as the water particles pass through the sheet, which enable the off-body flow to be visualized. However, the tunnel operator/owner might become perturbed at putting water in the tunnel. To alleviate his/her fear and to remove the water from the tunnel after the vapor screen runs, pump the tunnel down to a  $P_{\text{total}}$  of 500 psf, start the tunnel(subsonic) and turn the driers on. Do this approximately twice for 15 minutes each time. This will evaporate a majority of the water. After doing this have the tunnel technicians enter the tunnel and wipe up what

little water is remaining.

### Smoke Seeded Flow

Flow visualization using smoke is an excellent tool for observing off-body (external) flow fields that are dominated by vortical flows. If done correctly, superb qualitative data will result at a Reynolds number that might make it feasible to extrapolate the flow field to flight conditions.

Typically, smoke is generated by several methods. Such methods include pyrotechnic smoke devices, chemical (titanium tetrachloride and tin tetrachloride), petroleum products (kerosene, Type 1962 Fog Juice) and Rosco smoke generating fluid. The technique of producing smoke will be determined at the wind tunnel site.

A low turbulence wind tunnel is extremely helpful when trying to accomplish a flow visualization study using smoke. The surrounding air in the test section needs to be as undisturbed as possible in order that an accurate analysis of the model's flow field can be made.

Generally, the smoke enters the wind tunnel via a smoke wand that is typically located in the stilling chamber of the wind tunnel. The density of the smoke emitting from the wand should range from a mild to moderate fog (qualitatively speaking). The smoke should then traverse down to the test section and enter as a filament sheet that spans most of the section. The smoke filament sheet should be set (vertically) in such a manner that the smoke is "caught up" in the upwash of the wing, chine, or item of interest. It might be necessary to change the position of the the smoke wand to achieve an optimum smoke filament sheet position. Once the smoke filament sheet is caught in the upwash, the external flow field can be observed using a laser light sheet to expose the vortical systems.

### Tufts

To use tufts for off-body flow visualization, a tuft wand or a tufted, framed, wire grid is used. A tuft wand is a pole with a long tuft on the end of it. The wand is hand held in the tunnel near the model to observe off-body flow fields. This technique is a qualitative procedure since the person and the pole create a flow disturbance in the tunnel. A tufted, framed, wire grid gives the testing engineer a planar view of the off-body flow field. The wire grid contains symmetrical, square (1 inch by 1 inch ) wire mesh with tufts (usually yarn) glued at the corners of each mesh. The wire grid is placed



downstream of the model and is useful in examining wing tip vortices.

Determining Aerodynamic Angles from the Model Support (sting) Angles

Aerodynamic angles  $\alpha$  and  $\beta$  are calculated using the model support (sting) angles  $\gamma$ ,  $\psi$ ,  $\theta$ , and  $\phi$ .

Where

$\gamma$  = prebend angle of sting (only in pitch)

$\psi$  = yaw angle of the support

$\theta$  = pitch angle of the support

$\phi$  = roll angle of the support

Knowing

$$\alpha = \tan^{-1} \left( \frac{w}{u} \right)$$

$$\beta = \sin^{-1} \left( \frac{v}{\bar{V}} \right)$$

Where

$u$  = longitudinal velocity component

$v$  = lateral velocity component

$w$  = vertical velocity component

$\bar{V}$  = total freestream velocity

from reference 28 and 38

$$u = \left[ \cos\psi \left( \cos\gamma \cos\theta - \sin\gamma \sin\theta \cos\phi \right) - \sin\gamma \sin\phi \sin\psi \right] V_{\infty}$$

$$v = \left[ \sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi \right] V_{\infty}$$

$$w = \left[ \cos\psi \left( \sin\gamma \cos\theta + \cos\gamma \cos\phi \sin\theta \right) + \cos\gamma \sin\phi \sin\psi \right] V_{\infty}$$

Solving for the model angles  $\alpha$  and  $\beta$

$$\tan\alpha = \frac{\left[ \cos\psi \left( \sin\gamma \cos\theta + \cos\gamma \cos\phi \sin\theta \right) + \cos\gamma \sin\phi \sin\psi \right]}{\left[ \cos\psi \left( \cos\gamma \cos\theta - \sin\gamma \sin\theta \cos\phi \right) - \sin\gamma \sin\phi \sin\psi \right]}$$

$$\sin\beta = [\sin\phi\sin\theta\cos\varphi - \cos\phi\sin\varphi]$$

$$\phi = 0$$

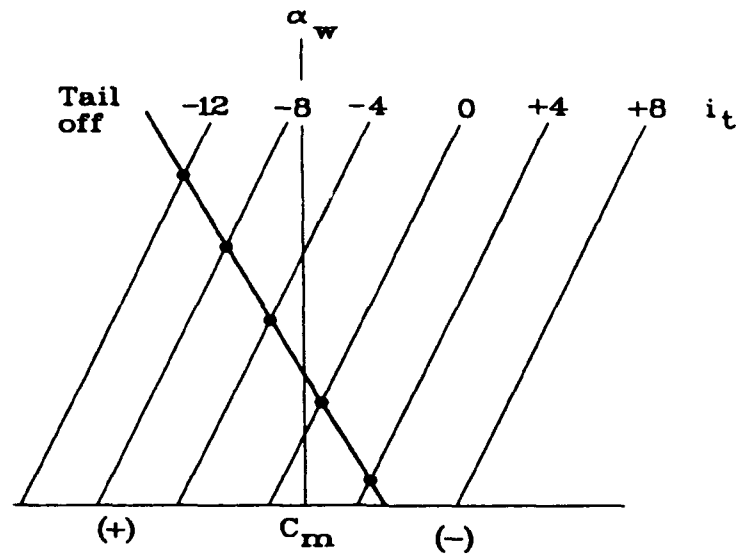


Figure VIII-1 Downwash Determination

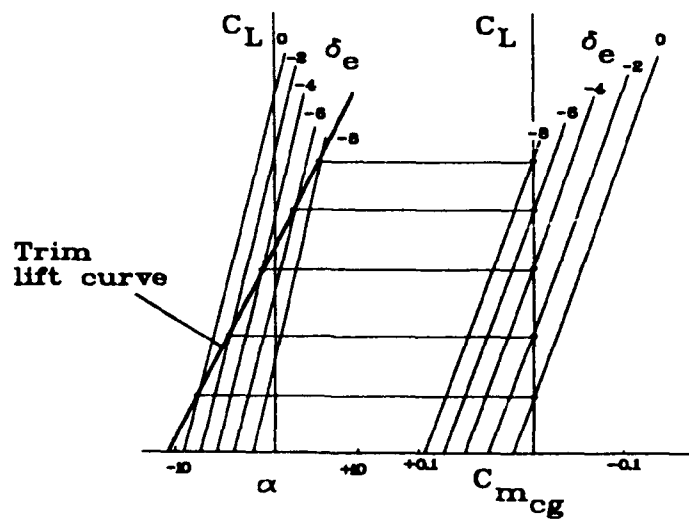


Figure VIII-2 Trim Determination Plot

## This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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## NOTES

[illegible]

## NOTES

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## NOTES

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## IX STRESS ANALYSIS

## IX Stress Analysis

### List of Symbols

A	Area
d	Diameter
c	Distance from Neutral Axis
E	Modulus of Elasticity
$f_{tu}$	Ultimate (allowable) Tensile Stress
$f_{su}$	Ultimate (allowable) Stress in pure shear
$f_{ty}$	Tensile Yield Stress (point)
$f_{tp}$	Tensile Proportional limit
G	Modulus of Rigidity (Shear Modulus of Elasticity)
I	Moment of Inertia
J	Polar Moment of Inertia
L	Total length of element (shaft length etc...)
l	Length of element (not shaft length)
$\Delta L$	Change in length after deformation
M	Moment
NF	Normal Force
P	Load
R	Average radius
r	Distance to point of interest for torsion
$r_o$	Shaft radius
$\Delta s$	Change in arc length after deformation
T	Torque
t	Thickness
y	Distance to point of interest for stress
$\alpha$	Constant based on $l/t$ (non dimensional)
$\beta$	Constant based on $l/t$ (non dimensional)
$\epsilon$	Strain
$\sigma$	Stress
$\tau$	Shear

List of Symbols continued

$\mu$	Poisson's ratio
$\gamma$	Angle of shear strain

Subscripts

i -- inside	b -- bending	n -- normal
o -- outside	s -- shear	

## Definitions

<u>Stress</u>	Stress implies a force per unit area and is a measure of the intensity of the force acting on a definite plane passing through a given point. The stress distribution may or may not be uniform, depending on the nature of the loading conditions. Tensile load is considered (+) and a compressive load is considered (-).
<u>Strain</u>	It is the change in length per unit length. The strain distribution may or may not be uniform depending on the member and loading conditions.
<u>Normal Stress</u>	A unit stress which acts normal to the cross section of the structural element. These stresses are created by bending moments and axial forces.
<u>Shear Stress</u>	A unit stress which acts parallel and in the plane of the cross section. These stresses are caused by torsional moments and shear forces.
<u>Normal Strain</u>	Strain associated with a normal stress; it takes place in the direction in which its associated normal stress acts. Increase in length are (+) strains, decrease in lengths are (-) strains.
<u>Shearing Strains</u>	Those strains related to relative changes in angles.
<u>Yield Point</u>	Where elongation increases with no increase in load. The stress at this point is known as Tensile Yield Stress.
<u>Proportional Limit</u>	Where the stress-strain curve first becomes non-linear. That point is the maximum stress where the strain remains directly proportional to stress.
<u>Elastic Limit</u>	The maximum stress to which a material may be subjected and still upon the removal of the load, return to its original dimensions.
<u>Neutral Axis</u>	When a beam is deflected, one surface is in compression while the other surface is under tensile stress. There is a plane (neutral) where the stress will be zero. Where this neutral

<u>Neutral Axis</u> (continued)	plane intersects any perpendicular cross section (or plane of loading) this location is the neutral axis for that cross section.
<u>Ultimate Tensile Stress</u>	This is the maximum allowable stress of the material.
<u>Factor of Safety</u>	Ultimate load / limit load
<u>Modulus of Elasticity</u>	Ratio of stress to strain; slope of the straight portion of the stress-strain diagram.
<u>Shear Modulus of Elasticity</u>	Ratio of shear stress to shear strain at low loads. The initial slope of the stress-strain diagram for shear.
<u>Radius of Gyration of an Body</u>	The distance from the inertia axis that the entire mass would be concentrated in order to give the same moment of inertia.
<u>Radius of Gyration of an Area</u>	The distance from the inertia axis to the point where the area would be concentrated in order to produce the same moment of inertia.
<u>Shear Center</u>	A point where a load produces no torsion on a asymmetric beam cross section.

## STRESS FORMULAS

### Normal Stress

$$\sigma_n = \frac{P}{A}$$

### Normal Strain

$$\epsilon_n = \frac{\Delta L}{L}$$

### Modulus of Elasticity

$$E = \frac{\sigma_n}{\epsilon_n}$$

### Poisson's Ratio

$$\mu = \frac{\text{lateral deformation}}{\text{axial deformation}}$$

### Shear Strain (thin wall, circular cylinder: twcc)

$$\epsilon_s = \frac{\Delta s}{L}$$

### Shear Modulus of Elasticity (twcc)

$$G = \frac{\tau_s}{\gamma} = \frac{\sigma_s}{\epsilon_s}$$

## ANGLE OF TWIST

### Rectangular shaft

$$\phi = \frac{TL}{\beta I G t^3}$$

### Circular shaft

$$\phi = \frac{TL}{JG}$$

### Split tube

$$\phi = \frac{3TL}{2\pi R t^3 G}$$

## POLAR MOMENT OF INERTIA

### Solid, Circular Shaft

$$J = \frac{\pi d^4}{32}$$

### Tubular, Circular cross-section

$$J = \frac{\pi}{32} (d_o^4 - d_i^4)$$



Solid, Circular Shaft

$$J = \frac{\pi}{2} r^4$$

Tubular, Circular cross-section

$$J = \frac{\pi}{2} (r_o^4 - r_i^4)$$

RADIUS OF GYRATION

Body

$$\rho = \sqrt{\frac{I}{M}}$$

Area

$$\rho = \sqrt{\frac{I}{A}}$$

BENDING STRESS

$$f_b = \sigma_x = \frac{Mc}{I}$$

$$f_b = \frac{6M}{bh^2} \text{ (rectangular)}$$

SHEAR STRESS

Rectangular cross section (beam)

$$f_s = \tau_{\max} = \frac{3P}{2A}$$

Circular cross section (beam)

$$f_s = \tau_{\max} = \frac{4P}{3A}$$

TORSIONAL FORMULAS

Solid, circular shaft

$$f_s = \tau_{\max} = \frac{16T}{\pi d^3} = \frac{Tr}{J}$$

Tubular, circular cross section

$$\begin{aligned} f_s = \tau_{\max} &= \frac{16\pi d_o}{\pi(d_o^4 - d_i^4)} \\ &= \frac{2\pi r_o}{\pi(r_o^4 - r_i^4)} \end{aligned}$$

### Split tube

$$f_s = \tau_s = \frac{3T}{2\pi R t^2}$$

### Rectangular cross section

(flat plate)

$$f_s = \tau_s = \frac{3T}{l t^2} ; l \gg t$$

$$f_s = \tau_s = \frac{T}{\alpha l t^2} ; l = t$$

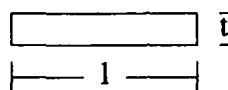


Table IX-1 Stress Constants (ref. 31)

l/t	1.0	1.5	1.75	2.0	2.5	3.0	4.0	6.0	8.0	10.0	$\infty$
$\alpha$	.208	.231	.239	.246	.258	.267	.282	.299	.307	.313	.333
$\beta$	.141	.196	.214	.229	.249	.263	.281	.299	.307	.313	.333

### COMBINED STRESS

Shafts under bending and torsional loads (ref. 33)

$$f_T = \frac{f_b}{2} + \sqrt{\left(\frac{f_b}{4}\right)^2 + f_s^2}$$

Stresses on an incline plane (ref. 34)

$$\sigma = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

### Principle Stress

$$\sigma_{\max \min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Angle on which principle stresses occurs  
(measured from x axis)

$$\tan 2\theta = \frac{-\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

Maximum and Minimum Shear stresses

$$\tau_{\max \min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Angle on which max and min Shear stress occurs  
(measured from x axis)

$$\tan 2\theta = \frac{\frac{\sigma_x + \sigma_y}{2}}{\tau_{xy}}$$

### FACTOR OF SAFETY

$$SF = \frac{f_{tu}}{\text{stress}}$$

$$SF = \frac{f_{tu}}{f_T}$$

$$SF = \frac{1}{\sqrt{\left(\frac{f_b}{f_{tu}}\right)^2 + \left(\frac{f_s}{f_{tu}}\right)^2}}$$

Calculate Centroid of a Planform Area

Elem.	Dim (in)	Area (in <sup>2</sup> )	$\bar{y}$ (in)	$A\bar{y}$ (in <sup>3</sup> )	$A\bar{y}^2$ (in <sup>4</sup> )
1	1.45 X 4.20	6.1	10.3+2.1= 12.4	75.5	937.9
2	2.3 X 10.3	23.7	10.3/2 = 5.14	122.0	628.6
3	8.98 X 2.0 X 0.5	8.98	8.98/3= 3.0	26.9	80.82
4	8.98 X 2.0 X 0.5	8.98	8.98/3= 3.0	26.9	80.82
5	0.83 X 2.0	1.66	0.83/2= .42	0.7	0.29
6	0.83 X 2.0	1.66	0.83/2= .42	0.7	0.29
7	0.85 X 4.22/2	1.8	4.22/3+10.3= 11.7	21.1	246.4
8	0.85 X 4.22/2	1.8	4.22/3+10.3= 11.7	21.1	246.4
		$\Sigma$ 54.7 in <sup>2</sup>		$\Sigma$ 294.9	$\Sigma$ 2221.5

$$\bar{y} = \frac{\Sigma A\bar{y}}{A} = \frac{294.9}{54.7} = 5.4 \text{ in}$$

$$I_X = 2221.5 \text{ in}^4$$

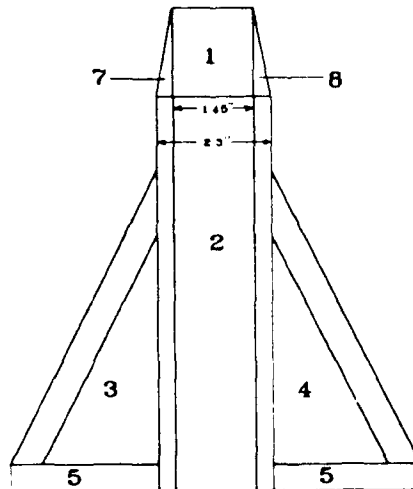


Figure IX-1      Stress Analysis Example 1  
(centroid determination)

### Example: Stress Analysis

Shear and bending at section F-F (figure IX-2)

Assuming a rectangular section of average thickness...

$$f_b = \frac{6M}{l t^2} ; \quad \begin{array}{l} l = \text{base} \\ t = \text{height (thickness)} \end{array}$$

$$(f_b = \frac{Mc}{I}; I = \frac{lt^3}{12}; c = \frac{t}{2})$$

$$f_s = \frac{T}{\alpha l t^2}$$

T = torsion in beam

$\alpha$  = constant dependent upon  $l/t$

$$l = 0.84 \text{ in}$$

$$t_{\text{ave}} = \frac{0.044 + 0.030}{2} = 0.037 \text{ in}$$

$$l/t = 22.7 ; \alpha = 0.333$$

$$M = 6.0(0.380) = 2.28 \text{ in-lb}$$

$$T = 6 \left[ 0.473 + \frac{0.84}{2} \right] = 5.358 \text{ in-lb}$$

### Bending

$$f_b = \frac{6(2.28)}{(0.840)(0.037)^2} = 11,896 \text{ psi}$$

$$\text{S.F.} = \frac{89100}{11896} = 7.48$$

### Shear

$$f_s = \frac{5.358}{(0.333)(0.840)(0.037)^2} = 13,992 \text{ psi}$$

$$S.F. = \frac{1}{\sqrt{\left[\frac{11896}{89100}\right]^2 + \left[\frac{13992}{57000}\right]^2}} = 3.57$$

Shear and bending at first set of attachment holes at section E-E  
(figure IX-2)

Assuming a rectangular section of average thickness...

$$f_b = \frac{6M}{l t^2} ;$$

$$(f_b = \frac{Mc}{I}; I = \frac{lt^3}{12}; c = \frac{t}{2} )$$

$$f_s = \frac{T}{\alpha l t^2}$$

T = torsions in beam

$\alpha$  = constant dependent upon  $l/t$

$$l = 0.840 - 2(0.138) = 0.564 \text{ in}$$

$$t_{ave} = \frac{0.062 + 0.030}{2} = 0.046 \text{ in}$$

$$l/t = 12.3 ; \alpha = 0.333$$

$$M = 6.0(0.380 + 0.210) = 3.54 \text{ in-lb}$$

$$T = 6 \left[ 0.473 + \frac{0.84}{2} \right] = 5.358 \text{ in-lb}$$

### Bending

$$f_b = \frac{6(3.54)}{(0.564)(0.046)^2} = 17,798 \text{ psi}$$

$$\text{S.F.} = \frac{89100}{17798} = 5.01$$

### Shear

$$f_s = \frac{5.358}{(0.333)(0.564)(0.046)^2} = 13,482 \text{ psi}$$

$$\text{S.F.} = \frac{1}{\sqrt{\left[ \frac{17798}{89100} \right]^2 + \left[ \frac{13482}{57000} \right]^2}} = 3.23$$

Tension in screws at wing tip missile attachment...

Summing moments about A-A

$$\Sigma M_{A-A} = 0 = 1.57NF - 0.706R_1 - 0.558R_2 - 0.154R_3 - 0.302R_4$$

Solving screw loads in terms of  $R_1$

$$R_2 = \frac{0.558}{0.706}R_1$$

$$R_3 = \frac{0.154}{0.706}R_1$$

$$R_4 = \frac{0.302}{0.706}R_1$$

Substitute into the  $M_{A-A}$  equation above...

$$1.57NF = R_1 \left[ 0.706 + \frac{0.558^2 + 0.154^2 + 0.302^2}{0.706} \right]$$

$$1.57NF = 1.31R_1$$

$$R_1 = 1.199NF$$

From above...

$$R_2 = 0.948NF$$

$$R_3 = 0.262NF$$

$$R_4 = 0.308NF$$

From figure IX-2,  $NF = 6.0$  lbs...

$$R_1 = 7.2 \text{ lbs}$$

$$R_2 = 5.69 \text{ lbs}$$

$$R_3 = 1.57 \text{ lbs}$$

$$R_4 = 3.08 \text{ lbs}$$

$f_{tu}$  for 0-80 flat-head socket screws is 265 lbs

$$S.F. = \frac{265}{7.2} = 36.8$$

Thread pullout in wing tip missile attachment screws...

For screw  $R_1$  (0-80 flat-head screws)...

$$\text{Shear Area} = \pi(\text{screw pitch dia.})(\text{screw length})(0.5)$$

(0.5) is an arbitrary fudge factor for conservatism

$$= \pi(0.519)(0.58)(0.5)$$

$$\text{Shear Area} = 0.0094(0.5) = 0.0047 \text{ in}^2$$

Shear

$$f_s = \text{load} / \text{shear area}$$

$$f_s = 7.2 / 0.0047$$

$$f_s = 1532 \text{ psi}$$

For flat-head screw 0-80

$$f_{su} = 96000 \text{ psi}$$



$$\text{S.F.} = \frac{96000}{1532} = 62.7$$

For the wing: 17-4PH stainless steel screws...

$$f_{su} = 120000 \text{ psi}$$

$$\text{S.F.} = \frac{120000}{1532} = 78.4$$

Screw head pullout in wing tip missile attachment...

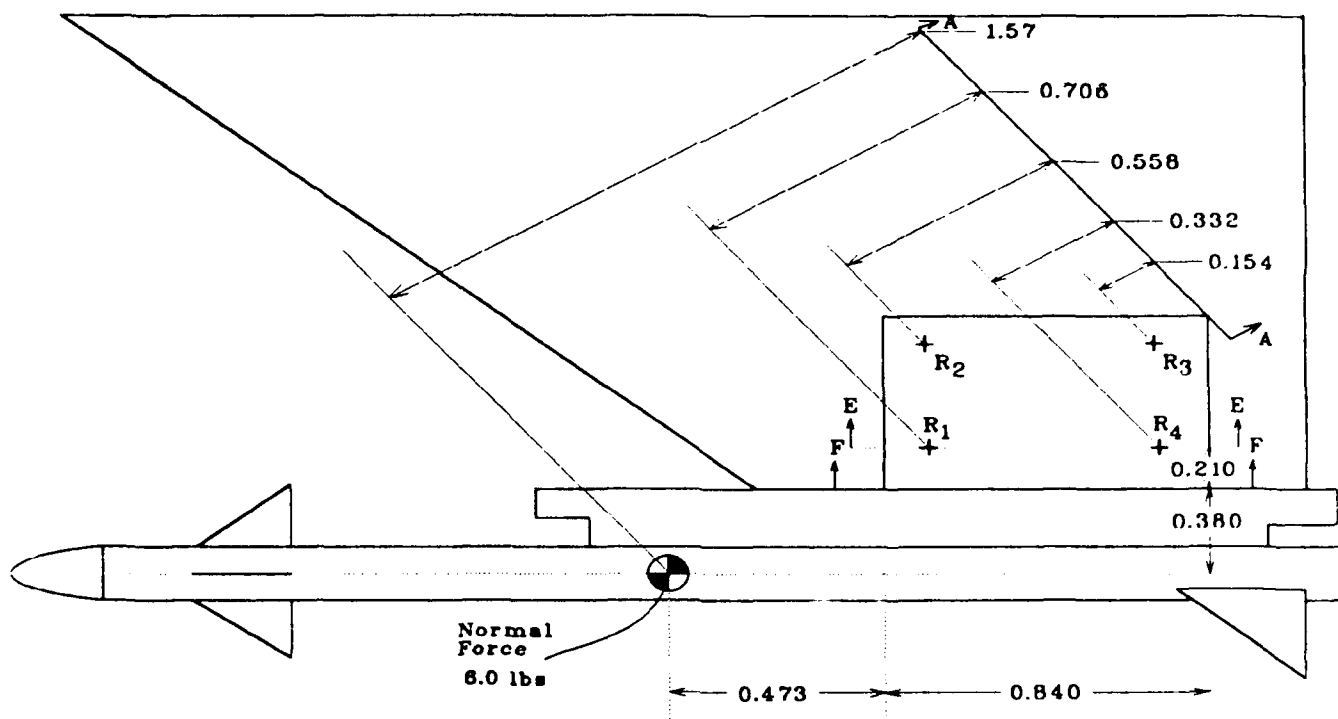
$$\text{shear area} = \pi(\text{screw head dia.})\left(\frac{1}{2} - \text{head depth}\right)$$

$$= \pi(0.117)(0.058 - 0.045) = 0.00478 \text{ in}^2$$

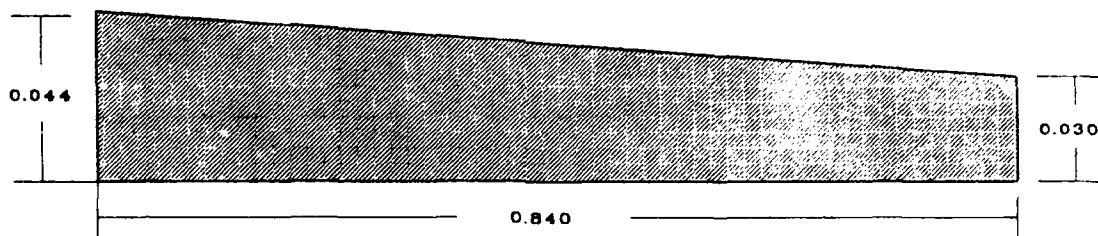
Shear force...(for 4130 steel)

$$f_s = \frac{7.2}{0.00478} = 1506.3 \text{ psi}$$

$$\text{S.F.} = \frac{89100}{1506.3} = 59.2$$



Section F-F



Section E-E

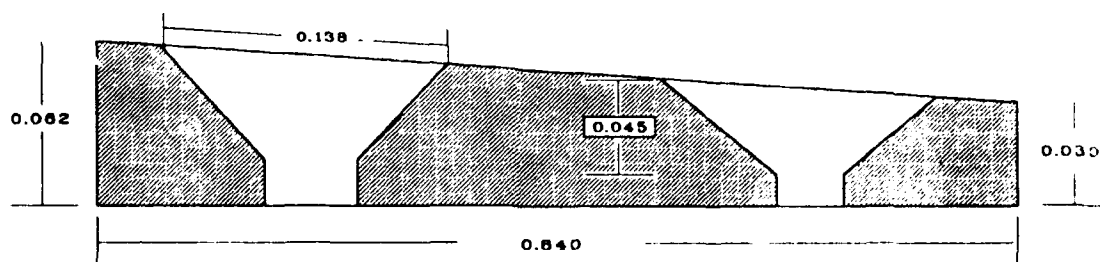


Figure IX-2 Stress Analysis Example 2  
(wing tip missile)

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## NOTES

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X TRENDS

X



# X Trends

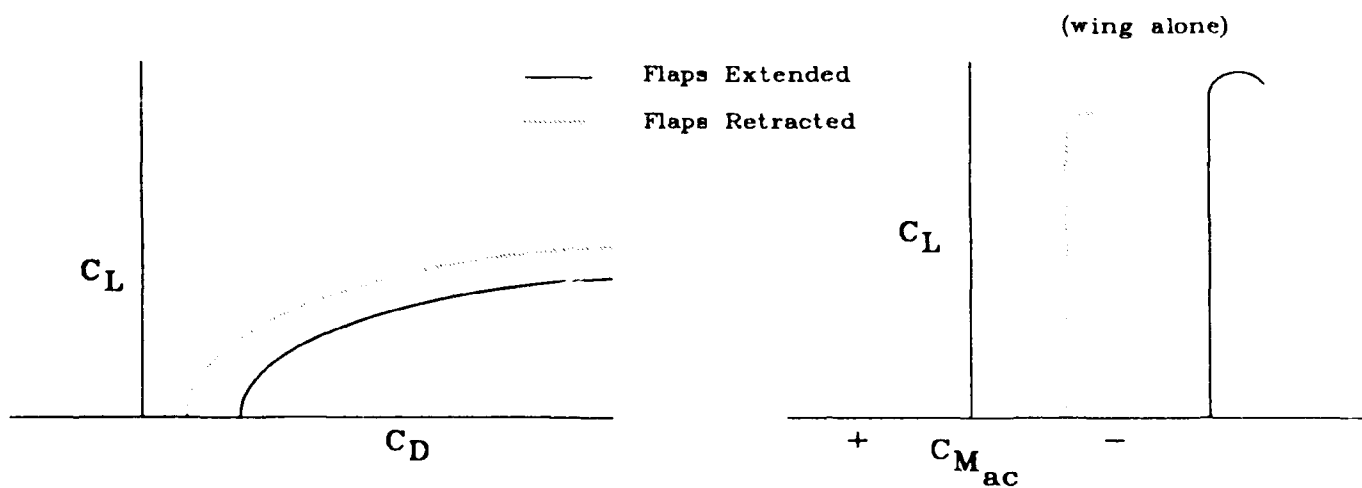
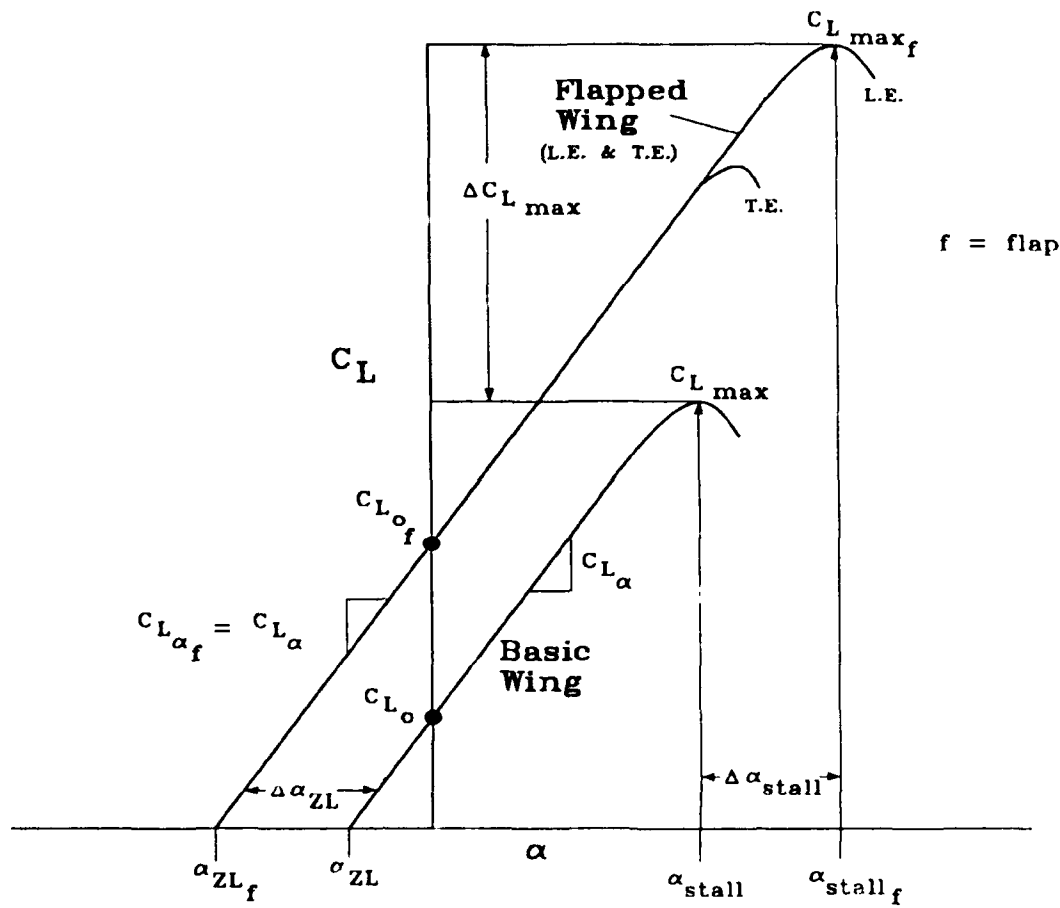


Figure X-1 Flap Characteristics

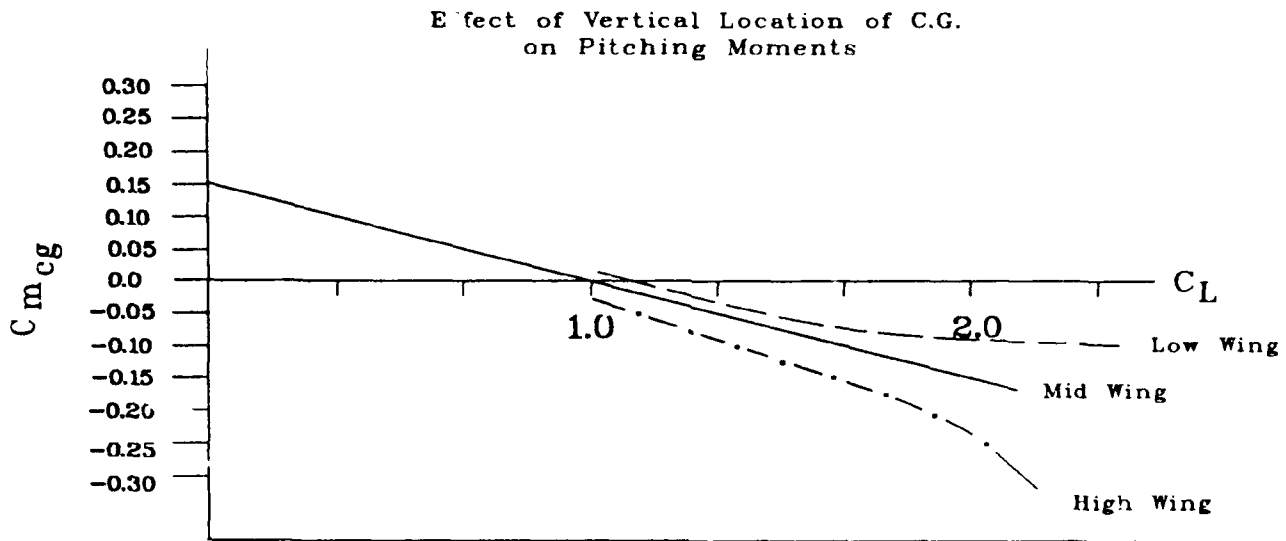


Figure X-2 Effect of Vertical Location of C.G. on Pitching Moments

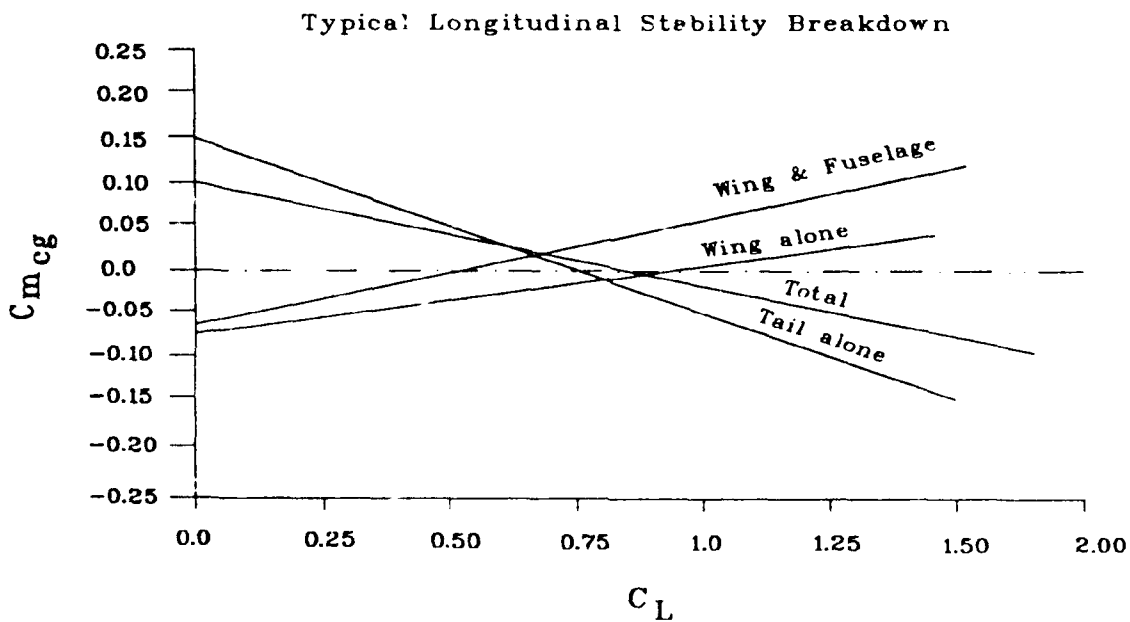


Figure X-3 Typical Component Longitudinal Stability Breakdown

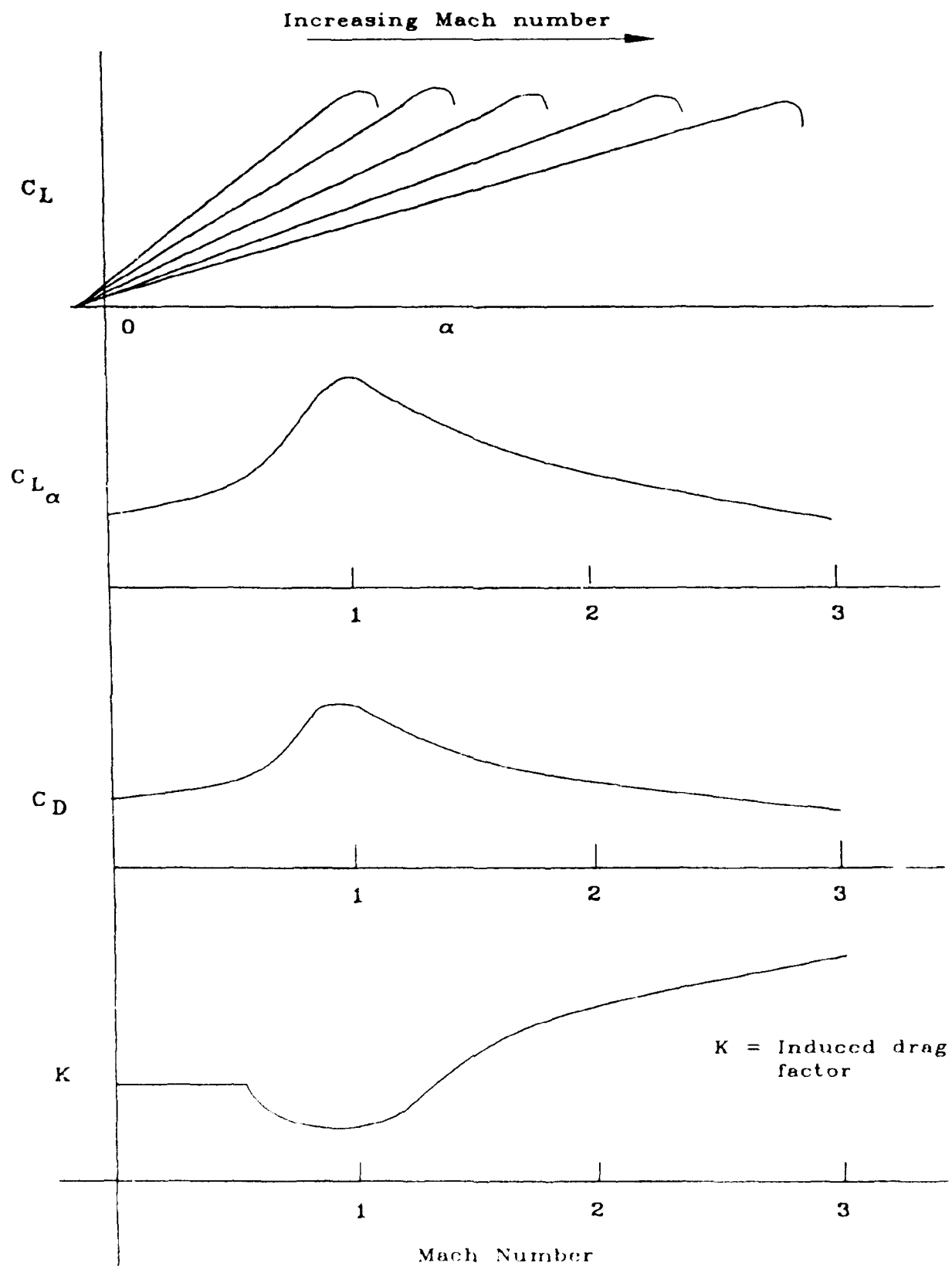


Figure X-4 Mach Number Trends (Effects)

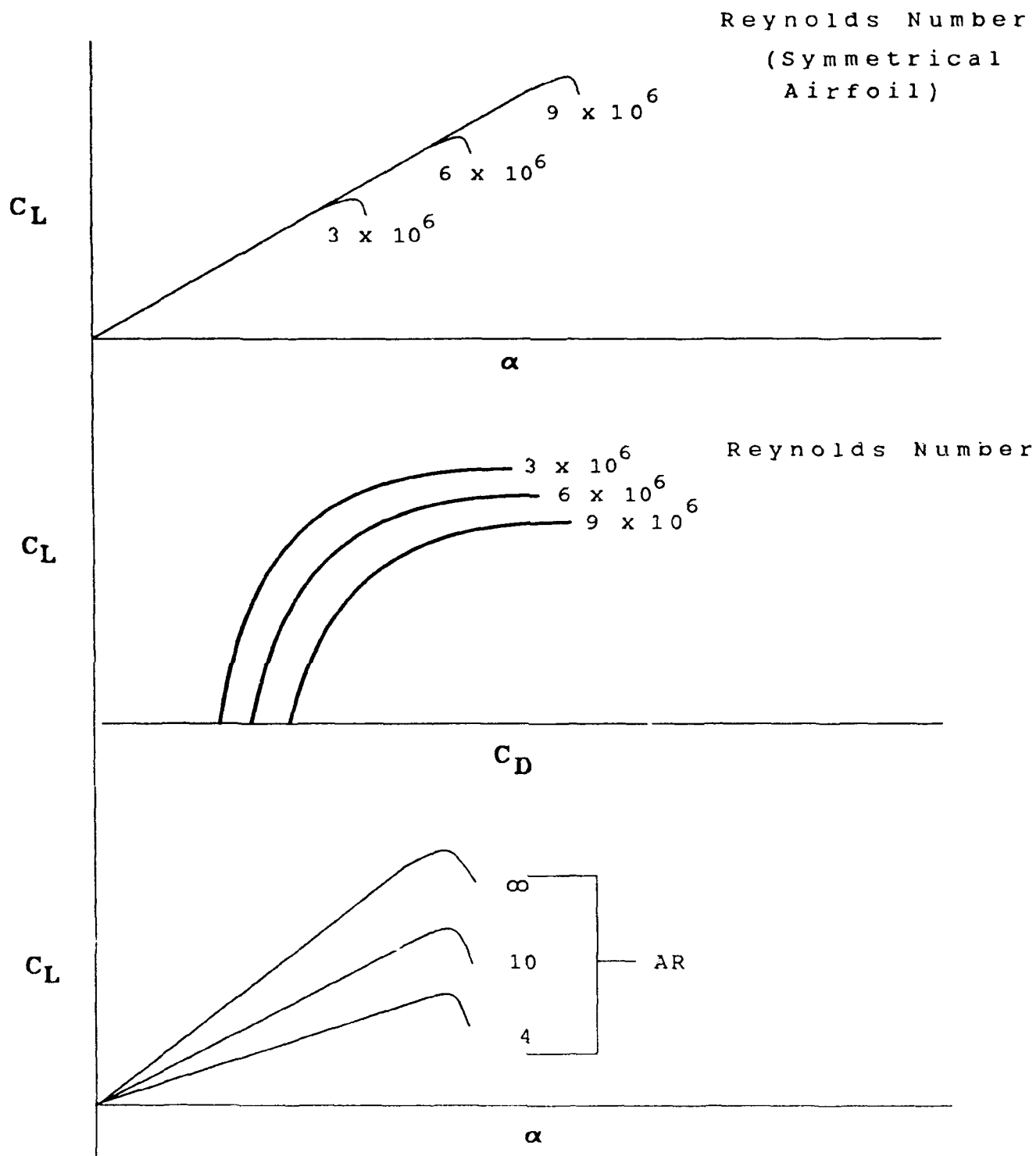


Figure X-5 Reynolds Number and Aspect Ratio Trends (Effects)

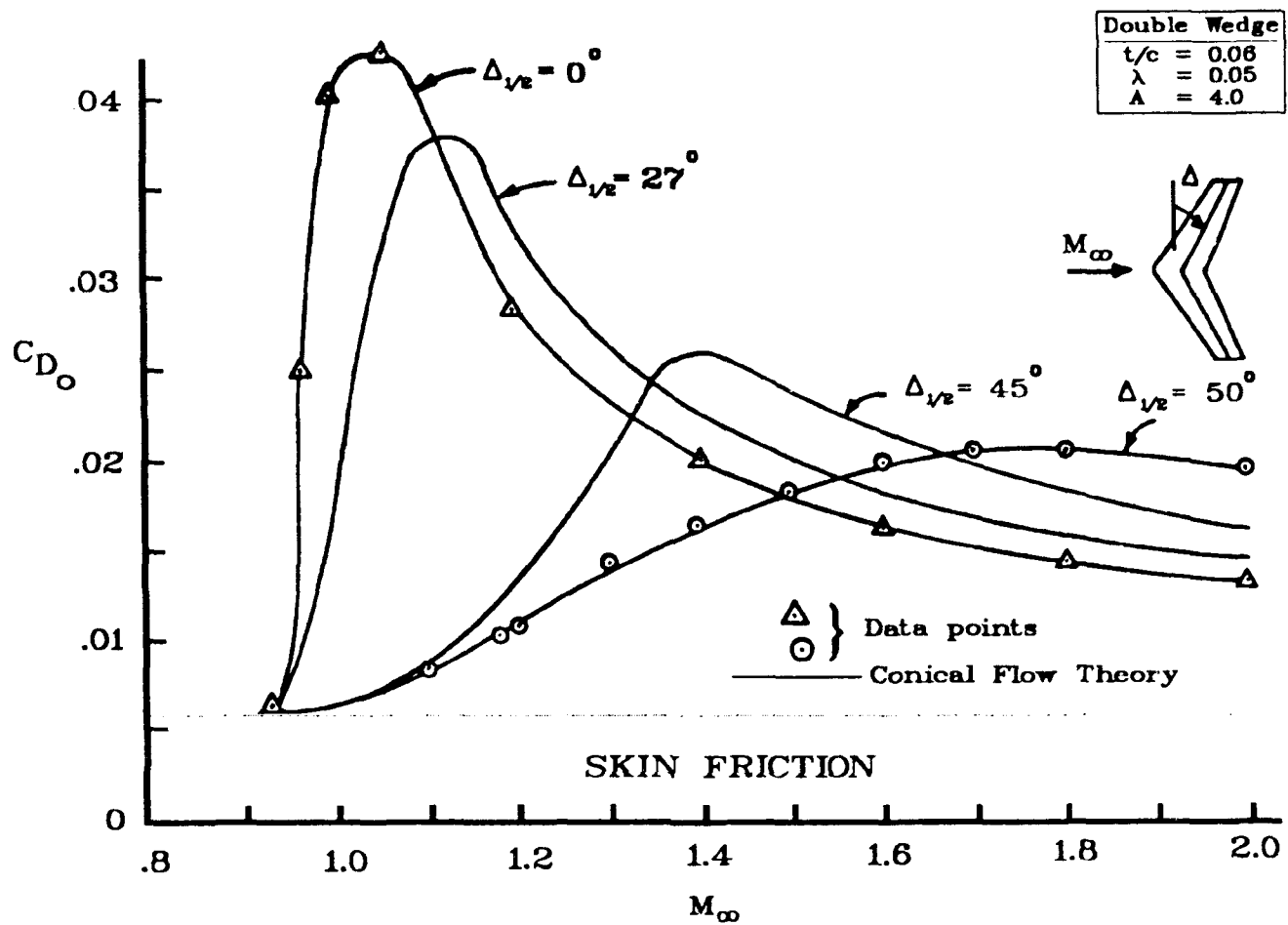
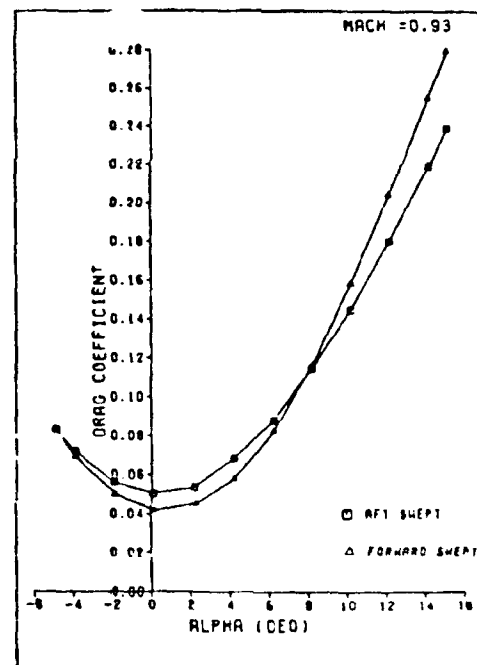
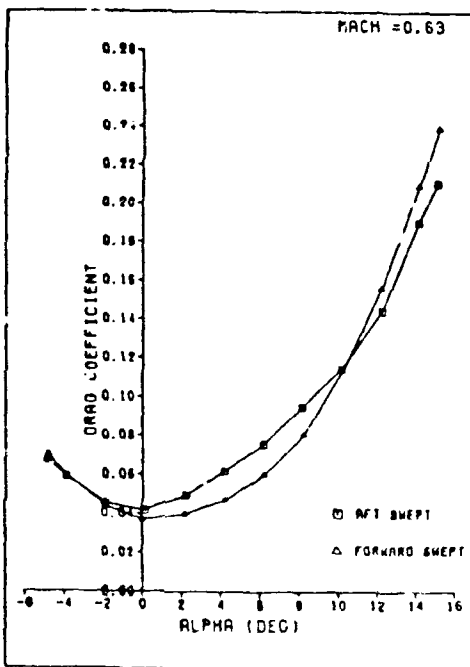
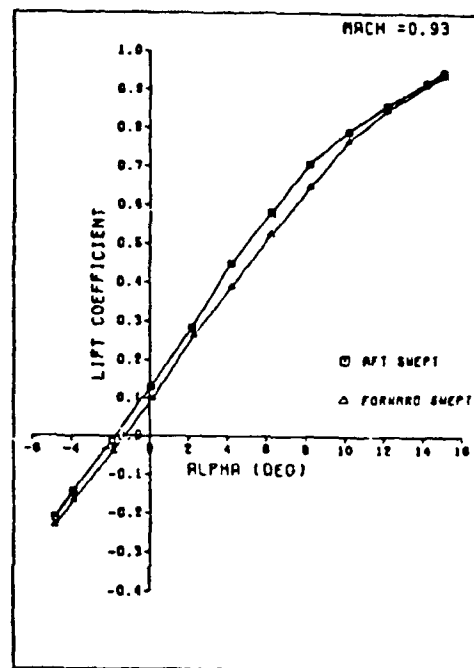
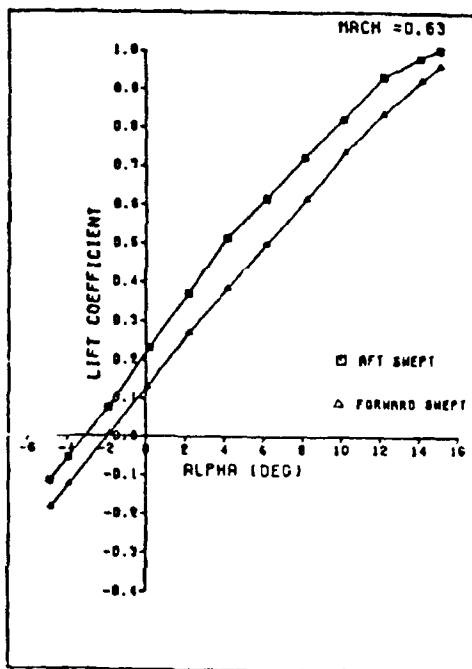


Figure X-6 Effect of Wing Sweep on  $C_{D_0}$



$$AR = 3.34$$

$$\Lambda_{C/4} = \pm 45^{\circ}$$

$$S_w = 25.7 \text{ in}^2$$

$$MAC = 5.62''$$

$$\lambda = 0.4$$

$$t/c = 5.5\%$$

$$b/2 = 7.58''$$

$$C_r = 6.1''$$

$$C_L = 2.45''$$

Figure X-7 Aft and Forward Swept Wing-Fuselage Effects  
(Ref. 18)

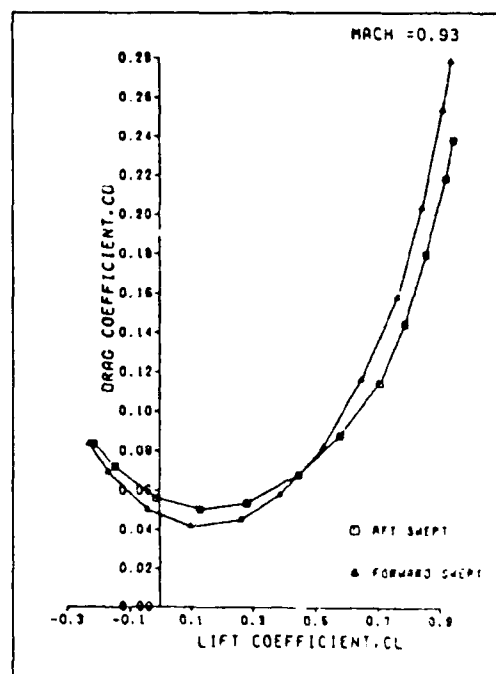
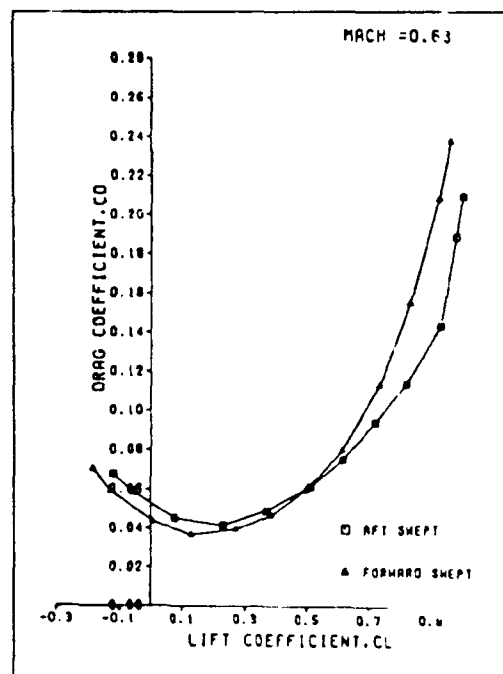
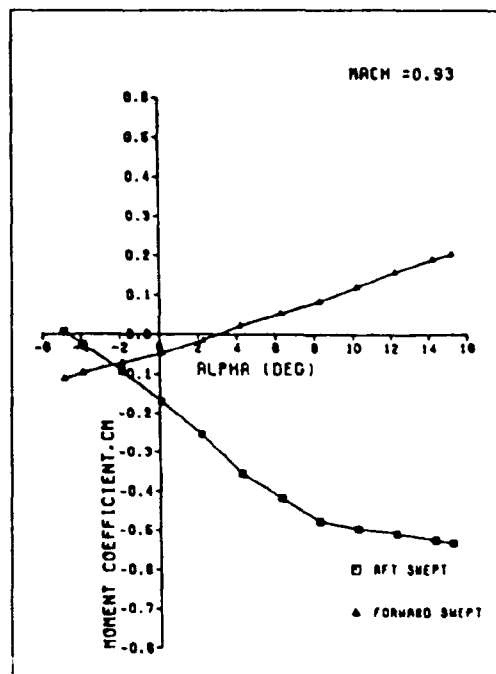
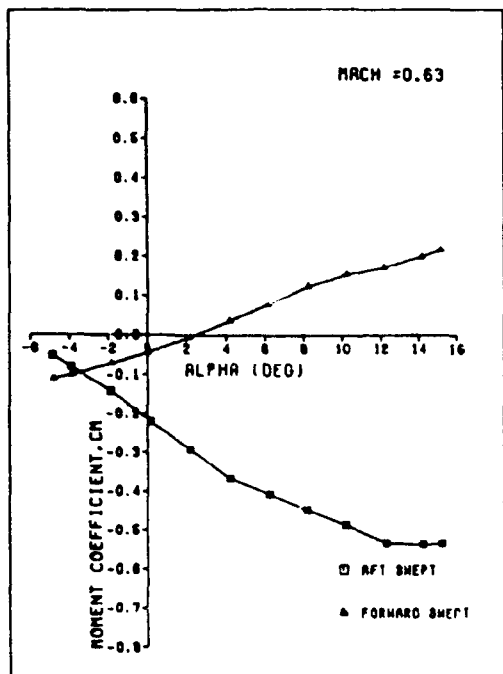


Figure X-7 continued

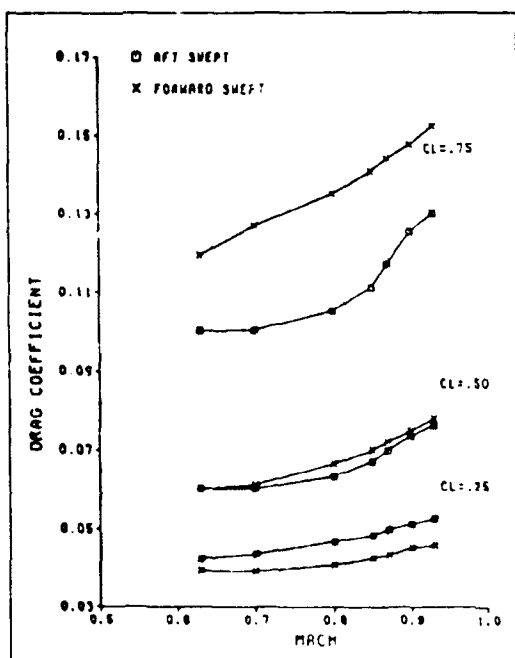
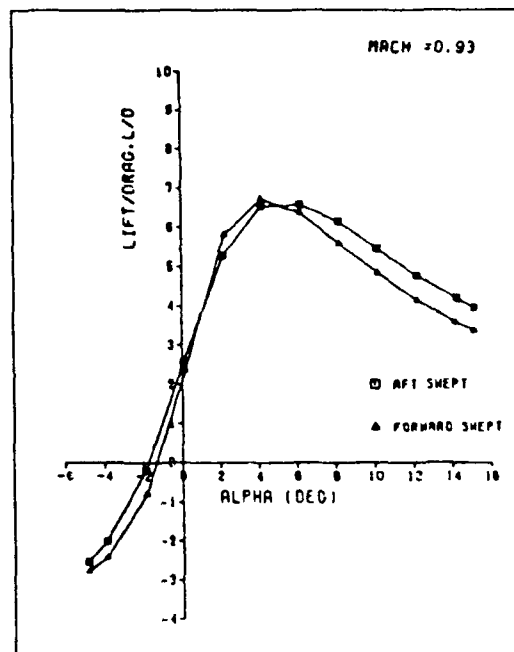
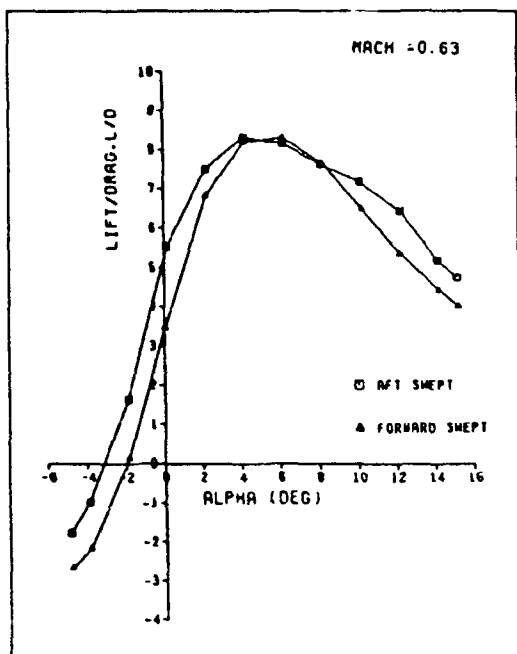
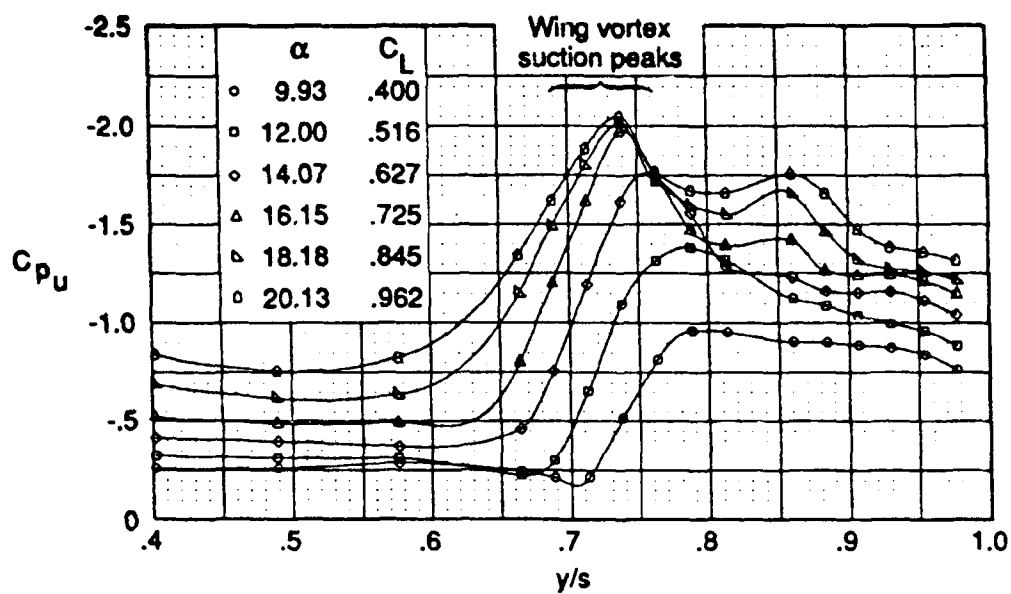
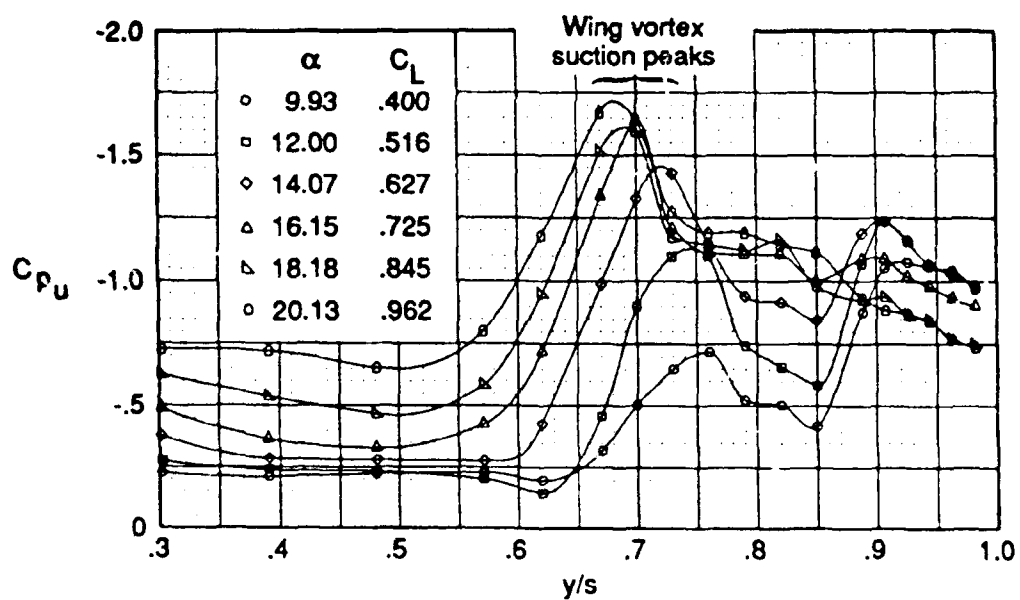


Figure X-7 concluded



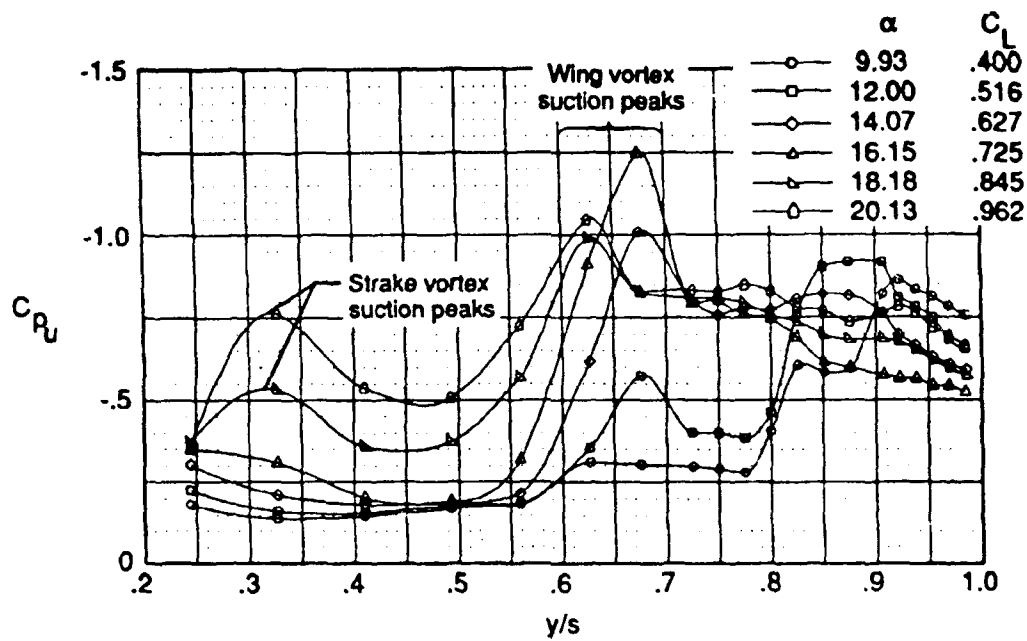


(a)  $x/c=0.50$



(c)  $x/c=0.62$

Figure X-8 Wing Pressure Distribution in the presence of a Coupled Chine (ref. 19)



(e)  $x/c=0.75$

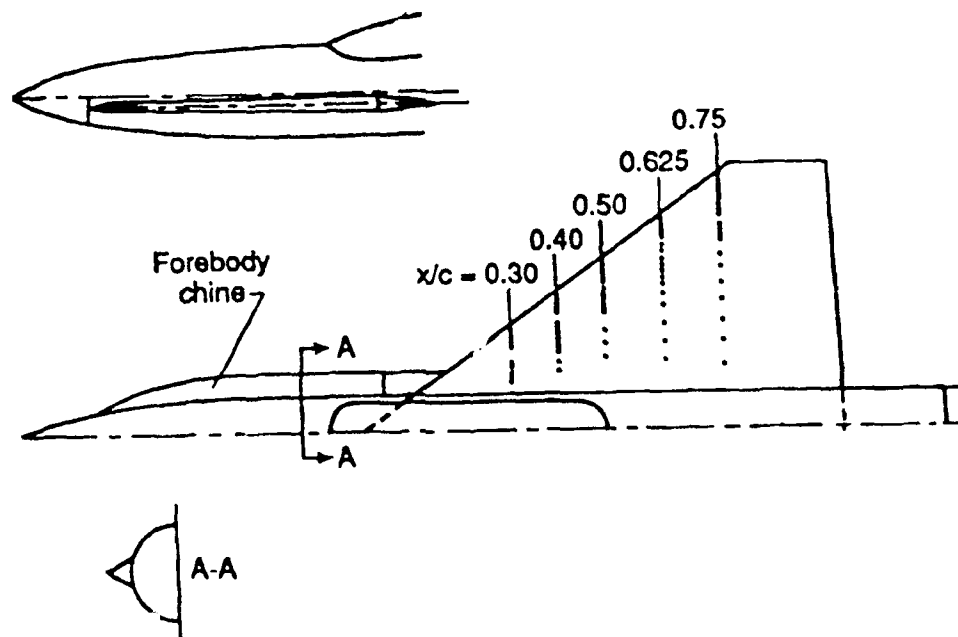


Figure X-8 concluded

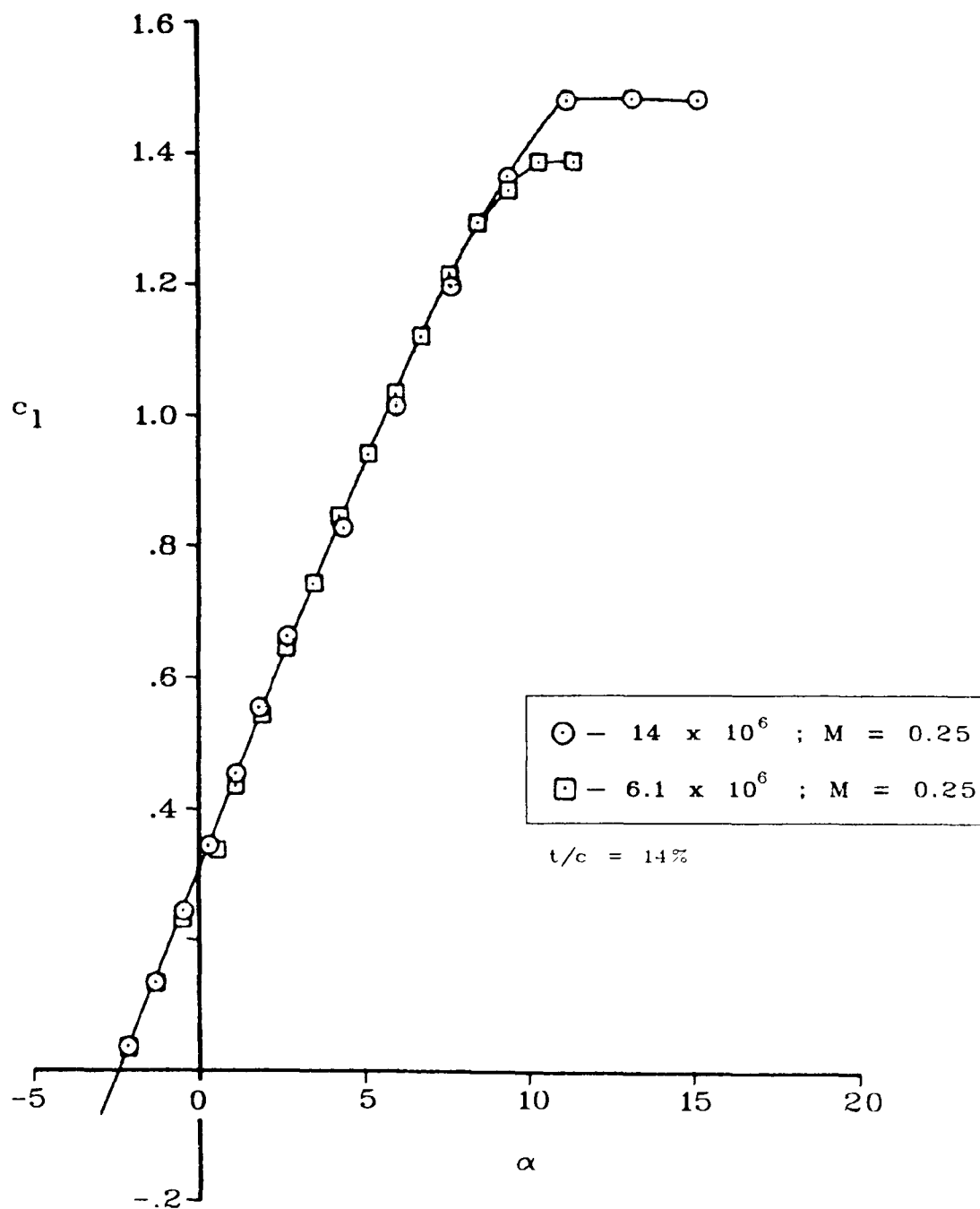


Figure X-9 Reynolds Number Effect on  $c_{l \max}$   
(wing alone)

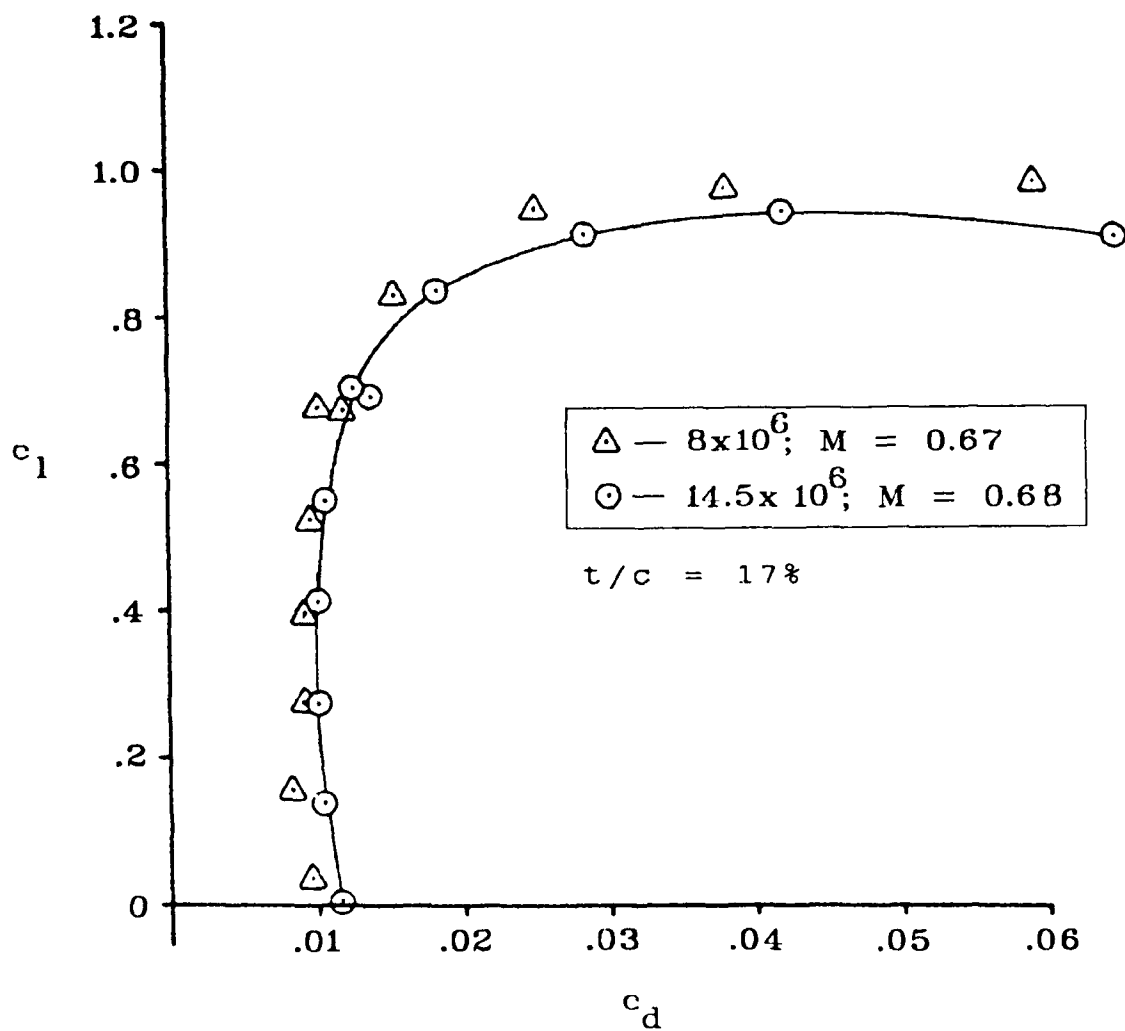


Figure X-10 Reynolds Number Effect on Drag  
(wing alone)

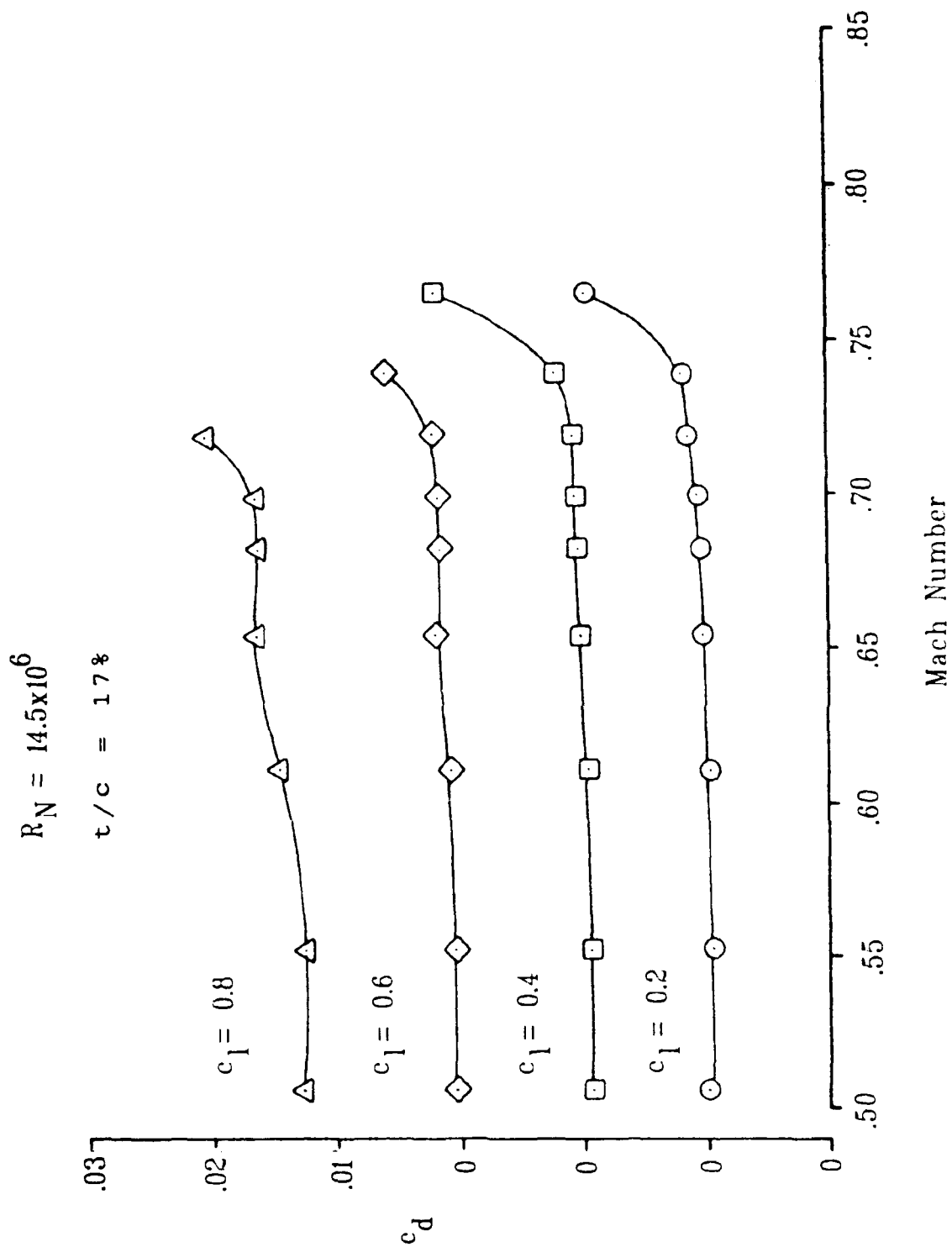


Figure X-11 Drag Rise Characteristics  
(wing alone)

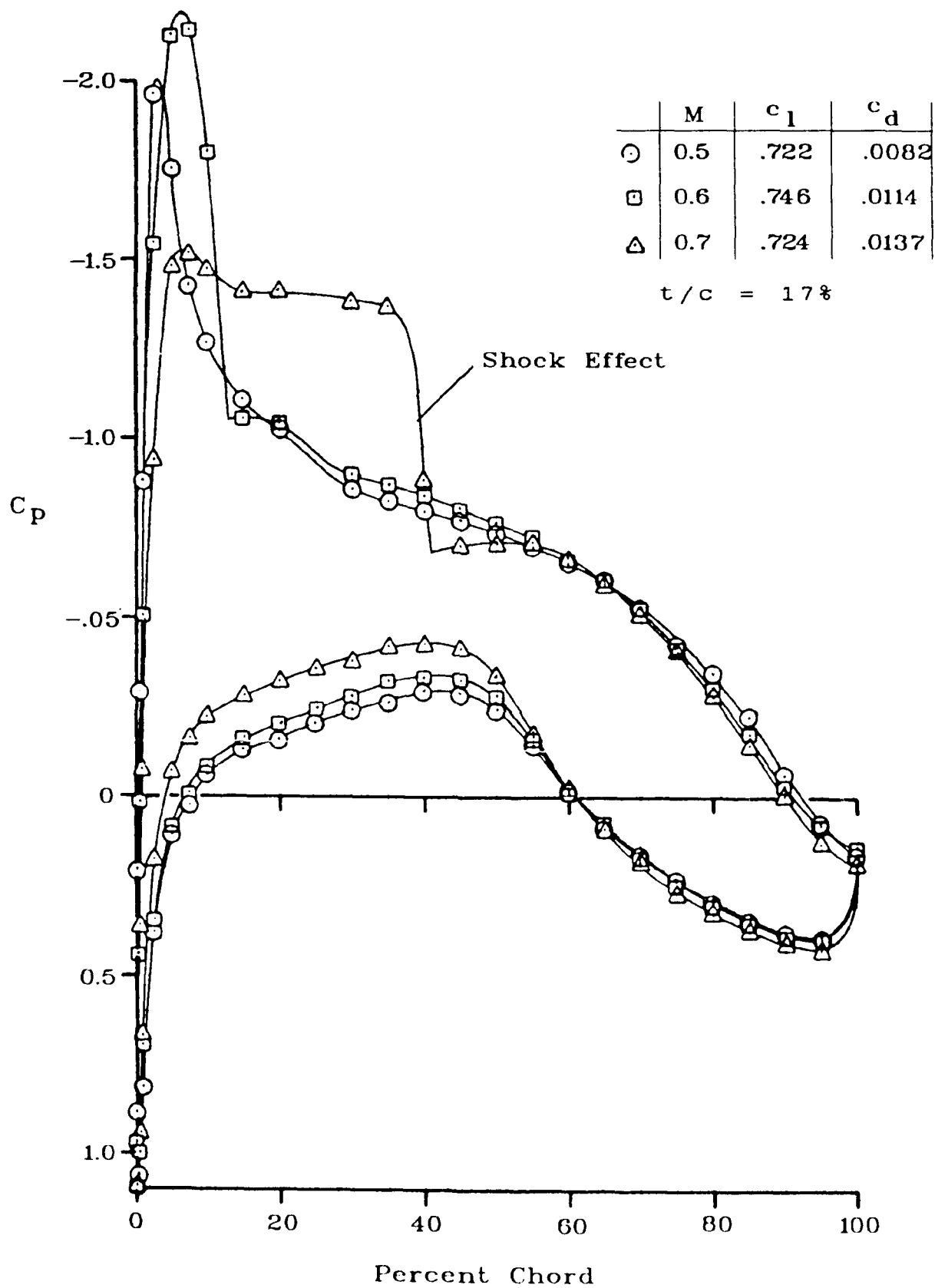


Figure X-12 Mach Effect on Airfoil Pressure Distribution

## NOTES

[illegible]

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## XI INTERNAL STRAIN GAGE BALANCES

## XI Internal Strain Gage Balances

(Ref. 36)

### List of Symbols

A	Cross-sectional area
E	Voltage
F	Gage Factor
$\{F_A\}$	Applied load to balance matrix
$\{F_o\}$	Output (mv) force matrix
$[K_{ij}]^{-1}$	Inverted balance calibration matrix
L	Length
l	Rolling moment
m	Pitching moment
mv	Milli-volts
n	Yawing moment
R	Resistance
(SC)	Sensitivity constant matrix
wt	Weight
X	Axial force
Y	Side force
Z	Normal force
$\epsilon$	Axial (local) strain
$\Delta$	Incremental
$\mu$	Poisson's ratio
$\rho$	Resistivity of the gage material

### Subscripts

A - Applied load	AF - Axial force	NF - Normal force
O - Output (mv)	PM - Pitching moment	SF - Side Force
RM - Rolling moment	YM - Yawing moment	

## Strain Gages

In strain measurement, the simplest device to use is the resistance type strain gage. Its construction and operation are simple, but it is so precise that strains on the order of 0.1% may be measured. The gage is about the size of a postage stamp but only slightly heavier. It consists of a metallic wire or strip of foil whose electrical resistance varies linearly with strain. The gage is securely bonded to the member to be strained so that any strain in the member due to a load is transmitted to the wire. There are literally hundreds of types of strain gages available commercially; each gage having been developed in response to a demand for a gage to meet a specific condition.

The most fundamental part of the gage is the wire itself. The resistance increase of a wire, when it is stretched, is due to an increase in the length and a decrease in its cross section, than to actual change in specific resistance. Typically, an electrical conductor (wire) is bonded to the specimen (balance) with an insulating cement under a no-load condition. A load is then applied, which produces a deformation in both the specimen and resistance element (wire). This deformation is indicated through a measurement of the change in resistance of the element (wires).

Three common types of resistance strain gages used in internal balances are wire, foil, and semi-conductor. The bonded wire is most commonly used with the wire diameter varying between 0.005 and 0.001 inches. The foil gage usually employs a foil less than 0.001 inches thick. The semi-conductor gage employs a silicon base material that is strain sensitive and has the advantage of a very large gage factor ( $F \sim 100$ ). (The gage factor relates the electrical resistance to the physical properties of the gage.) The material is usually produced in brittle wafers having a thickness of 0.01 inches.

Wire and foil gages may be manufactured in various ways, but the important point is that the resistance element (wire/foil) be securely bonded to its mounting. Most wire strain gages employ either a nitrocellulose cement or a phenolic resin for the bonding agent with a thin paper backing to maintain the wire's configuration. Such gages may be used up to 300°F. A Bakelite mounting is usually employed for temperatures up to 500°F. Foil gages are manufactured by an etching process and use base material of paper, Bakelite, and epoxy film. Epoxy cement is also used on wire and foil gages.

## Temperature Effects

The major source of strain gage error is the fact that the resistance of most wires changes with temperature. This variation is not only a function of the change in temperature but, may also be a function of the number of heating cycles to which the gages has been subjected and of the time elapsed between cycles. Temperature compensation may easily be accomplished by installing a second strain gage, often known as a dummy gage, on an unstrained piece of the same metal that the active gage is bonded to. There are two changes in gage resistance which are caused by temperature changes. The first effect is the temperature coefficient of resistivity which is a random effect that may be considered as independent of the gage factor. The second effect is the difference in linear expansion between the gage and the member (material) on which it is mounted. The latter effect is one in which the error in the strain gage output is significant. Semi-conductor gages offer the advantage that they have a lower expansion coefficient then either wire or foil gages. It's best to make sure the strain gages are temperature compensated.

## Deformation Theory and Calculation

Basic relations for the resistance strain gage are

$$R = \rho \left( \frac{L}{A} \right) \quad (1)$$

$L$  = length

$A$  = cross-sectional area

$\rho$  = resistivity of the material

Differentiating (1)

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad (2)$$

The area may be related to the square of some transverse dimension, such as the diameter ( $D$ ) of the resistance wire.

$$\frac{dA}{A} = 2 \left( \frac{dD}{D} \right) \quad (3)$$

The unit axial strain  $\epsilon$  is defined as

$$\epsilon = \frac{dL}{L} \quad (4)$$

Poisson's ratio is defined as

$$\mu = \frac{dD/D}{dL/L} \quad (5)$$

Substituting equations (3), (4) and (5) into (2)...

$$\frac{dR}{R} = \epsilon(1 + 2\mu) + \frac{d\rho}{\rho} \quad (6)$$

The gage factor 'F' is defined as

$$F = \frac{dR/R}{\epsilon} \quad (7)$$

Substituting (7) into (6)

$$F = 1 + 2\mu + \left(\frac{1}{\epsilon}\right)\left(\frac{d\rho}{\rho}\right) \quad (8)$$

Rearranging (7)

Local Strain

$$\epsilon = \frac{1}{F} \left( \frac{\Delta R}{R} \right) \quad (9)$$

The value of the gage factor 'F' and the resistance (R) are usually specified by the manufacturer so that the user need only measure the change of resistance ( $\Delta R$ ) in order to determine the local strain due to a load.

#### Measurement of $\Delta R/R$

Consider the basic Wheatstone bridge in figure XI-1. The voltage at the detector is given by

$$E_D = E \left[ \frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right] \quad (10)$$

If the bridge is balanced then  $E_D = 0.0$  volts.

Let the strain gage represent  $R_1$  in the circuit in figure XI-2 and a voltage readout is used such that the bridge operates as a voltage sensitive circuit. We assume that the bridge is balanced at zero strain conditions and that a strain,  $\epsilon$ , on the gage results in the change in the resistance  $\Delta R_1$  and a change of voltage  $\Delta E$  on the bridge.  $R_1$  will be used to represent the resistance of the gage at zero strain conditions.

The voltage due to strain is...

$$\frac{\Delta E_D}{E} = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_4} - \frac{R_2}{R_2 + R_3} \quad (11)$$

Solving for the resistance change...

$$\boxed{\frac{\Delta R_1}{R_1} = \frac{(R_4/R_1)[\Delta E_D/E + R_2/(R_2 + R_3)]}{1 - \Delta E_D/E - R_2/(R_2 + R_3)} - 1} \quad (13)$$

#### Balance Calibration (interaction) Matrix

(ref. 8 and 37)

No internal balance is able to measure the pure loads that it was intended to measure. This is due to errors within the balance. There are generally two types of errors. The first type of 'balance' errors are the linear (first degree) errors. This type of errors are primarily due to construction errors, improperly positioned gages, variation in gage factors, and electrical circuits. The second type of error is the non-linear (2nd degree) error. This error is primarily attributed to elastic deformation (deflections) of various balance parts. Both types of errors create interaction of forces and moments.

To account for these interactions (errors) in the balance, the balance is calibrated. The purpose of this calibration is to acquire a set of interaction equations that can be used to determine the loads applied by the model through the balance output (voltage) signals. This type of calibration will not (generally) be accomplished by the testing



engineer. All the testing engineer will receive will be a sheet(s) of paper with the linear and non-linear interaction coefficients on it (figure XI-3). For a six component (3 forces and 3 moments) balance, the linear coefficients will be used as a 6x6 matrix with near unity (1) on the diagonal. The non-linear coefficients will be used as a 6x21. For a six component balance, there is a total of 27 interaction terms. Generally speaking, the 6x6 matrix will need to be inverted to be used in the data acquisition computer. This can be seen below in the theory of how the computer acquires force/moment data from a balance.

$$\{F_O\} = [K_{ij}]\{F_A\}$$

$$\{F_A\} = [K_{ij}]^{-1}\{SC\}\{F_O\} \quad (14)$$

Example:

Original Balance Calibration Matrix  
(non-dimensional)

$$\begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix} = \begin{bmatrix} 1.0000 & .0336 & -.1764 & -.0549 & .0059 & -.0004 \\ -.0169 & 1.0000 & 0.0000 & .0131 & .0000 & .0000 \\ -.0059 & -.0022 & 1.0000 & -.0032 & .0000 & .0002 \\ -.0004 & -.0006 & .0078 & 1.0000 & .1012 & -.0044 \\ .0000 & .0002 & .0000 & -.0412 & 1.0000 & -.0064 \\ .0012 & .0049 & .0294 & -.1555 & -.0384 & 1.0000 \end{bmatrix} \begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix}$$

### Inverted Original Balance Calibration Matrix

$$\begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix} = \begin{bmatrix} 1.0000 & -.0332 & -.176 & .0555 & -.0115 & .0005 \\ .0169 & .9994 & .0031 & -.0121 & .0011 & -.0000 \\ .006 & .002 & 1.001 & .0034 & -.0004 & -.0002 \\ .0004 & .0006 & -.0078 & .9964 & -.1007 & .0037 \\ .0000 & -.0002 & -.0005 & .0421 & .9959 & .0065 \\ -.0013 & -.0048 & -.0309 & .1564 & .0225 & 1.0008 \end{bmatrix} \begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix}$$

The interactions of the forces and moments can be seen in the inverted matrix. For an example, there is a -17.6% interaction on the normal force due to the the pitching moment and a 3.7% interaction on the rolling moment due to the side force.

The testing engineer needs to know if the balance coefficients are indeed non-dimensional or dimensional (ie...  $(\frac{\text{load}}{\text{volts}})$ ). If the interaction matrix is non-dimensional, then the testing engineer will have to acquire a sensitivity constant  $(\frac{\text{load}}{\text{volts}})$  for each force and moment. The acquisition of the sensitivity constants is acquired during the check loading of the balance (see page XI-10).

#### Calibration Body

In order to load a balance accurately, it is first necessary to construct a fixture. This fixture is generally called a calibration body (cal. body). The cal. body simulates the model to be tested which will allow loads to be accurately transmitted to the balance in the same manner as the model will during testing. It should be attached in the same manner as the model is attached to the balance (generally a single pin) and should have reference surfaces which will remain fixed with respect to the balance reference center (usually the balance electrical moment center)

The cal. body is an accurately machined 'rectangle box' with precisely located V-shaped grooves in which the loads are applied during calibration or check loadings. It is important that both the cal. body and the model be aligned with the balance in exactly the same manner. Also, the balance should fit tightly (no relative motion between the two items) in the cal. body and in the model. All fixtures (moment arms, weight pans etc...)

should be manufactured as light as possible and as stiff as possible.

Loads applied to the balance (via the cal. body) components are considered absolute loads. Therefore, the weight of the cal. body and fixtures and even parts of the balance must be considered as applied loads. However, the weight of the cal. body and fixtures are taken out in the 'zero' reading. The zero reading is the reference voltage to be subtracted out from the load reading to acquire a 'true' reading.

### Check Loading

The purpose of check loading a balance serves several purposes.

- 1) Acquire sensitivity constants (if needed)
- 2) Proof (check) load the balance
- 3) Check gaging and wiring
- 4) Determine hysteresis and repeatability
- 5) Determine component sensitivity
- 6) Determine deflections when a load is applied
- 7) Determine accuracy

It is important to check load the balance over the entire anticipated load and center-of-pressure range and that sufficient points are taken to assure accurate and repeatable loadings. The minimum check load to hang would be 10% of the balance limit.

### Check Loading Procedure

This procedure will assume that a dead weight (ie... disks) will be used to acquire the loadings required. However, a load is a load whether it is generated by a disk or hydraulic actuator or some other source. Just make sure it's a point load and not a distributed load on the calibration body.

Hang the weights on the calibration body at CONSTANT INCREMENTS, as an example,  $\Delta 50$  lbf or  $\Delta 100$  lbf or whatever weight increment you choose to use. Be sure to maintain that increment at all times. Don't forget to take a 'zero' voltage reading prior to hanging weights. A zero reading includes the weights of the cal. body, weight pan or any other weight that's considered a 'zero' weight. After loading a weight on the weight pan, stop the weight pan from swaying. Place an inclinometer on the cal. body and raise or lower the cal. body (via the sting) until it is level (or  $< \pm 3$  mins of a degree). This

takes out the weight component due to the deflection of the sting. Also, sting deflection measurements can be taken at this time. After the weight has been added and the swaying has been stopped, have the control room take a data point and record the output (normally in milli-volts [mv], and then add the next incremental weight and repeat the above procedure. Do this until the maximum load is achieved. Once the maximum load is applied and voltage taken, remove the weights at the same increment as when you added the weights and take a data point at each weight going down. Do this until all the weights have been removed and then take another zero. This up and down procedure of check loading will show accuracy, repeatability and a zero shift of the balance. Do this for all force and moment check loadings.

### Dead-Weight Loading System

When the dead-weight system is employed, the weights are generally in the form of cast iron disks (accurate to 0.1%) hanging on a weight pan supported by the cal. body or moment arm fixture. For a highly accurate check loading, one can weigh the cast iron disks prior to check loading.

When hanging weights for the normal force and side force, at a minimum, hang the weights at the balance electrical moment center and use this check loading to determine the normal and side forces sensitivity constant. Taking the sensitivity constant for the forces at the balance electrical moment center, allows the contribution (interactions) of the principle moments (pitching and yawing) to become negligible.

Hanging the weights for the pitching and yawing moments, at a minimum, at one moment arm ahead and behind the electrical moment center but at different moment arms. This will result in a positive and a negative moment. When check loading for the rolling moment, place the rolling moment fixture at the balance electrical moment center and at a minimum, apply loads at two different moment arms on the rolling moment fixture (expect to see a normal force reading).

### Axial Force Check Loading

Axial force check loading is generally accomplished with a pulley rig. A pulley rig has two wires that are connected to the cal. body that run down the side of the sting and over the pulleys down to a 'T' weight pan where the weights are placed. The 'T' weight pan must be absolutely level. Any angle in the 'T' weight pan will produce undesirable

interactions and an erroneous axial force will be acquired. Also, it might be desirable to check load in the negative axial force direction. If this is desirable, set the pulley rig in front of the cal. body and hang weights. Be sure to check the balance limits to see if a negative axial force limit exists. Normally, a negative axial force calibration is not done.

### Sensitivity Constants

Sensitivity constants are required when the balance calibration matrices are non-dimensional. They are acquired during check loading and are nothing but slopes of a line  $\left(\frac{\Delta \text{load}}{\Delta \text{volts}}\right)$  and are required for each force and moment. The sensitivity matrix is a diagonal matrix.

To acquire sensitivity constants...

- 1) Put balance calibration matrix in the data acquisition computer.
- 2) Initially set sensitivity constants to unity (1).
- 3) Accomplish the check loading in all planes and at multiple positions within that plane.
- 4) When the check loadings are complete, obtain the 'Raw Balance Data' (figure XI-4). Be sure and account for the zero reading. Usually it needs to be subtracted out of the output readings. There should be 'n' forces and moments usually titled X, Y, Z, l, m, n. Where X = axial force, Y = side force, Z = normal force, l = rolling moment, m = pitching moment, and n = yawing moment.
- 5) At this point use a linear regression program to acquire the slope of the line with the x-axis as the voltage and the y-axis as the load. The slope of this line is the sensitivity constant for that force or moment. Do this for each position and for each force and moment. The force and moment sensitivity constants should not vary much from one another when taken at different positions on that plane. For the force sensitivity constants, use the sensitivity constants at the balance electrical moment center. This eliminates, theoretically, the influence of the moment of that plane.
- 6) If a linear regression program is not available at the test site, take the first weight reading (mv) and subtract it from second weight reading (mv). Then take the second weight reading and subtract it from the third weight reading and so on. The

$\Delta$ milli-volts ( $\Delta$ mv) should be close to one another since the weight increment ( $\Delta$ wt) is held constant.

- 7) Obtain an average  $\Delta$ mv.
- 8) Divide the  $\Delta$ wt by the average  $\Delta$ mv. This is the sensitivity constant for that position on the plane.
- 9) Repeat steps 6-8 for each position on the plane.
- 10) Steps 6-8 are not as accurate as using a linear regression program on the computer.
- 11) For the moments, add your sensitivities together and divide by the number of sensitivities to obtain an average. This average sensitivity constant for the moment is the one given to the software/tunnel personnel.
- 12) After the sensitivity constants have been acquired, re-hang selected loads on the cal. body once again to verify the validity of the constants. The applied load to the output load should be within 0.5% of one another.

#### Example Sensitivity Constant Matrix

$$\begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix} = \begin{bmatrix} 2.3184 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.7844 & 0 & 0 & 0 & 0 \\ 0 & 0 & -.1867 & 0 & 0 & 0 \\ 0 & 0 & 0 & .1324 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.07780 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.188 \end{bmatrix} \begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix}$$

#### Obtaining Force and Moments from Raw Balance Data

- 1) Take the balance output (mv) and subtract out the zero reading (mv).
- 2) Take that increment ( $\Delta$ mv) and multiply it by that position on the balance calibration coefficient (ie... if you are checking the normal force, on the nxn matrix (which is usually inverted) go to the Z,Z position), and multiply it by its sensitivity constant. However, don't forget the interaction terms. They'll need to be added or subtracted to the force or moment you are looking at for an accurate reading.

$F$  or  $M = \Delta mv(\text{balance calibration})(\text{sensitivity constant}) + \text{interaction terms}$

$F$  = force;  $M$  = moment

Don't forget that each interaction term also has a sensitivity factor associated with it. Keep a close eye on the units. Don't mix apples and oranges.

#### Balance Calibration Equation Example (ref. 8)

The example below uses a 6 component balance calibration matrix. The calibration matrix below is considered non-dimensional. If the calibration matrix was dimensional, then consider the sensitivity constants on the diagonal of the sensitivity constant matrix as unity (1).

From equation 14

$$\{F_o\}(SC) = [K_{ij}]\{F_A\}$$

$$\{F_A\} = [K_{ij}]^{-1}(SC)\{F_o\}$$

#### Example Original Balance Calibration Matrix (non-dimensional)

$$\begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix} = \begin{bmatrix} 1.0000 & .0336 & -.1764 & -.0549 & .0059 & -.0004 \\ -.0169 & 1.0000 & .0000 & .0131 & .0000 & .0000 \\ -.0059 & -.0022 & 1.0000 & -.0032 & .0000 & .0002 \\ -.0004 & -.0006 & .0078 & 1.0000 & .1012 & -.0044 \\ .0000 & .0002 & .0000 & -.0412 & 1.0000 & -.0064 \\ .0012 & .0049 & .0294 & -.1555 & -.0384 & 1.0000 \end{bmatrix} \begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix}$$

### Example Inverted Balance Calibration Matrix

$$\begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix} = \begin{bmatrix} 1.0000 & -.0332 & -.176 & .0555 & -.0115 & .0005 \\ .0169 & .9994 & .0031 & -.0121 & .0011 & -.0000 \\ .006 & .002 & 1.001 & .0034 & -.0004 & -.0002 \\ .0004 & .0006 & -.0078 & .9964 & -.1007 & .0037 \\ .0000 & -.0002 & -.0005 & .0421 & .9959 & .0065 \\ -.0013 & -.0048 & -.0309 & .1564 & .0225 & 1.0008 \end{bmatrix} \begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix}$$

### Example Sensitivity Constant Matrix

$$\begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix} = \begin{bmatrix} 2.3184 & 0 & 0 & 0 & 0 & 0 \\ 0 & .7844 & 0 & 0 & 0 & 0 \\ 0 & 0 & -.1867 & 0 & 0 & 0 \\ 0 & 0 & 0 & .1324 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.0778 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.188 \end{bmatrix} \begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix}$$

A typical load (normal force) equation can be seen below.

$$Z_A = Z_O K_{11} SC_{NF} + X_O K_{12} SC_{AF} + Y_O K_{13} SC_{SF} + l_O K_{14} SC_{LM} + m_O K_{15} SC_{PM} + \dots$$

$$\dots + n_O K_{15} SC_{YM} \quad (15)$$

Using the inverted and sensitive constant matrix above, the total normal force (Z) and pitching moment (m) equations are listed below.

$$Z = Z(1)(2.3184) + X(-.0332)(-.7844) + m(-.176)(-.1867) + \dots$$

$$\dots + l(.0555)(.1324) + n(-.0115)(-.0778) + Y(.0005)(-1.188)$$

$$m = Z(.006)(2.3184) + X(.002)(-.7844) + m(1.001)(-.1867) + \dots$$

$$\dots + l(.0034)(.1324) + n(-.0004)(-.0778) + Y(-.002)(-1.188)$$

The rest of the forces and moments equations are derived in the same manner.



### Balance Placement in the Model

The physical placement of the balance in a wind tunnel model can vary. Usually, the balance electrical moment center is placed at the center of pressure of the configuration at the maximum aerodynamic load. Since the center of pressure position varies with angle-of-attack, make sure the aerodynamic load is not outside the load of the balance.

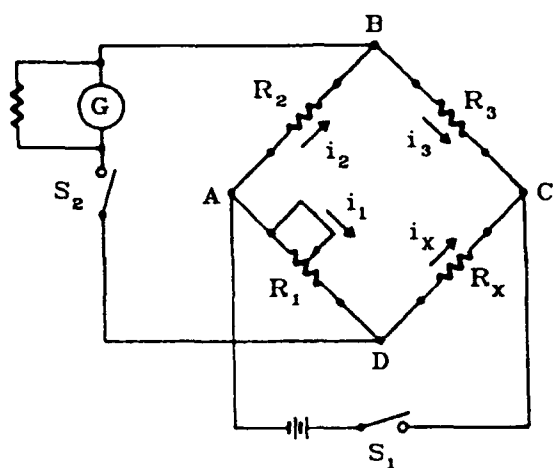


Figure XI-1 Basic Wheatstone Bridge

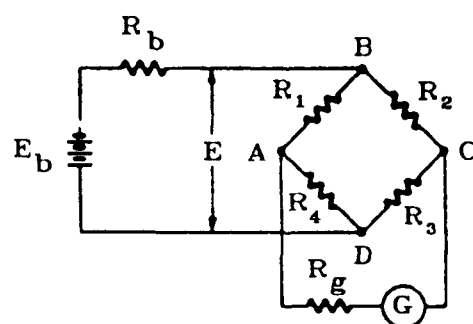


Figure XI-2 Unbalanced Wheatstone Bridge

# Normal Force

CARD SEQUENCE	COEFFICIENT SYMBOL	ENGLISH	VALUE	COMPONENT(S) OPERATED ON BY COEFFICIENTS
LINEAR INTERACTION COEFFICIENTS				
1	K(1)	-----	-----	NORMAL
2	K(2)	-1.6926E-02	-1.6926E-02	AXIAL
3	K(3)	-1.4902E-02	-5.8995E-03	PITCH
4	K(4)	-1.1272E-03	-4.4377E-04	ROLL
5	K(5)			YAW
6	K(6)	1.1570E-03	1.1570E-03	SIDE
NONLINEAR COEFFICIENTS				
7	K(7)			NORMAL SQUARED
8	K(8)			NORMAL X AXIAL
9	K(9)			NORMAL X PITCH
10	K(10)			NORMAL X ROLL
11	K(11)			NORMAL X YAW
12	K(12)	-5.7771E-07	-1.2987E-07	NORMAL X SIDE
13	K(13)			AXIAL SQUARED
14	K(14)	-5.1202E-07	-4.5319E-08	AXIAL X PITCH
15	K(15)			AXIAL X ROLL
16	K(16)			AXIAL X YAW
17	K(17)			AXIAL X SIDE
18	K(18)	4.0264E-08	1.4032E-09	PITCH SQUARED
19	K(19)			PITCH X ROLL
20	K(20)			PITCH X YAW
21	K(21)			PITCH X SIDE
22	K(22)	-3.0464E-08	-1.0617E-09	ROLL SQUARED
23	K(23)	-9.7235E-08	-3.3886E-09	ROLL X YAW
24	K(24)	-8.6340E-07	-7.6420E-08	ROLL X SIDE
25	K(25)			YAW SQUARED
26	K(26)			YAW X SIDE
27	K(27)	2.0256E-07	1.7929E-08	SIDE SQUARED

Figure XI-3 Example Balance Calibration Sheet

Normal Force (Z) Calibration  
(Balance Electrical Moment Center)

Wt Inc.	Data Pt	X	Y	Z	l	m	n
0	1	-1329.8	960.6	0.0	-1112.7	705.4	536.3
100	2	-1308.8	962.6	191.1	-1110.7	710.4	536.3
200	3	-1289.8	964.6	384.2	-1108.7	714.4	537.3
300	4	-1268.8	965.6	575.3	-1105.7	718.4	536.3
400	5	-1250.8	966.6	766.5	-1104.7	722.4	539.3
500	6	-1230.7	966.6	958.6	-1103.7	726.4	536.3
600	7	-1208.0	966.8	1150.9	-1101.9	731.6	534.4
700	8	-1186.7	967.6	1342.8	-1100.7	735.4	534.3
600	9	-1211.0	967.8	1149.9	-1102.9	730.6	536.4
500	10	-1227.7	964.6	957.6	-1103.7	725.4	536.3
400	11	-1250.8	966.6	765.5	-1106.7	721.4	536.3
300	12	-1268.8	965.6	573.3	-1107.7	727.4	536.3
200	13	-1287.8	965.6	381.2	-1110.7	713.4	538.3
100	14	-1306.8	964.6	189.1	-1112.7	709.4	540.3
0	15	-1326.8	962.6	0.0	-1113.7	704.4	541.3

Pitching Moment (m) Calibration  
(2 inch moment arm)

Wt Inc.	Data Pt	X	Y	Z	l	m	n
0	16	-1324.8	962.6	-2.0	-1113.7	688.4	543.3
100	17	-1310.8	963.6	182.1	-1110.7	567.3	544.3
200	18	-1298.8	964.6	369.2	-1108.7	446.3	545.3
300	19	-1286.8	964.6	555.3	-1105.7	326.2	545.3
400	20	-1271.8	965.6	742.4	-1102.7	206.1	545.3
500	21	-1260.8	965.6	928.6	-1100.7	85.1	546.3
600	22	-1248.7	966.6	1114.7	-1099.7	-36.0	545.3
700	23	-1233.7	966.6	1301.8	-1097.7	-157.1	545.3
600	24	-1247.7	965.6	1114.7	-1098.7	-35.0	547.3
500	25	-1256.8	966.6	928.6	-1100.7	86.1	545.3
400	26	-1273.8	965.6	742.4	-1102.7	208.1	545.3
300	27	-1287.0	965.8	556.4	-1103.9	329.3	545.4
200	28	-1298.0	964.8	370.3	-1106.9	450.4	545.4
100	29	-1309.8	964.6	183.1	-1108.7	571.3	547.3
0	30	-1322.8	963.6	0.0	-1110.7	692.4	547.3

All readings are in milli-volts (mv)

Wt Inc. is in lbf

Figure XI-4 Raw Balance Data

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## This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There is no text or other markings on the paper.



## NOTES

[illegible]

## LIST OF REFERENCES

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## APPENDIX A

## APPENDIX A

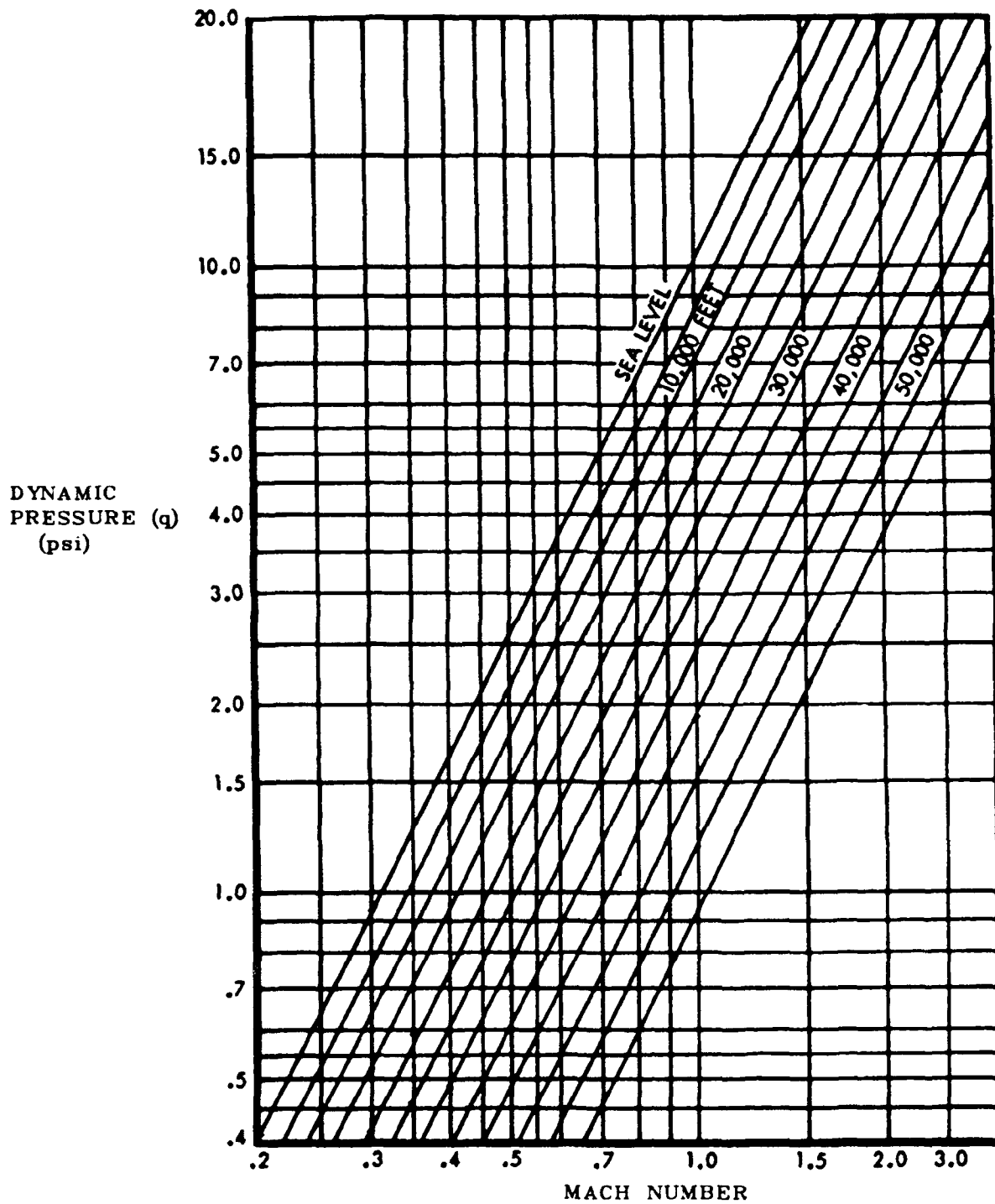


Figure A-1 Dynamic Pressure Determination  
(Standard Atmosphere)

Example:  
 $M = 0.25; T_t = 100^\circ$   
 $R/\rho = 0.11 \times 14.7$   
 $R/\rho = 1.6 \times 10^6$

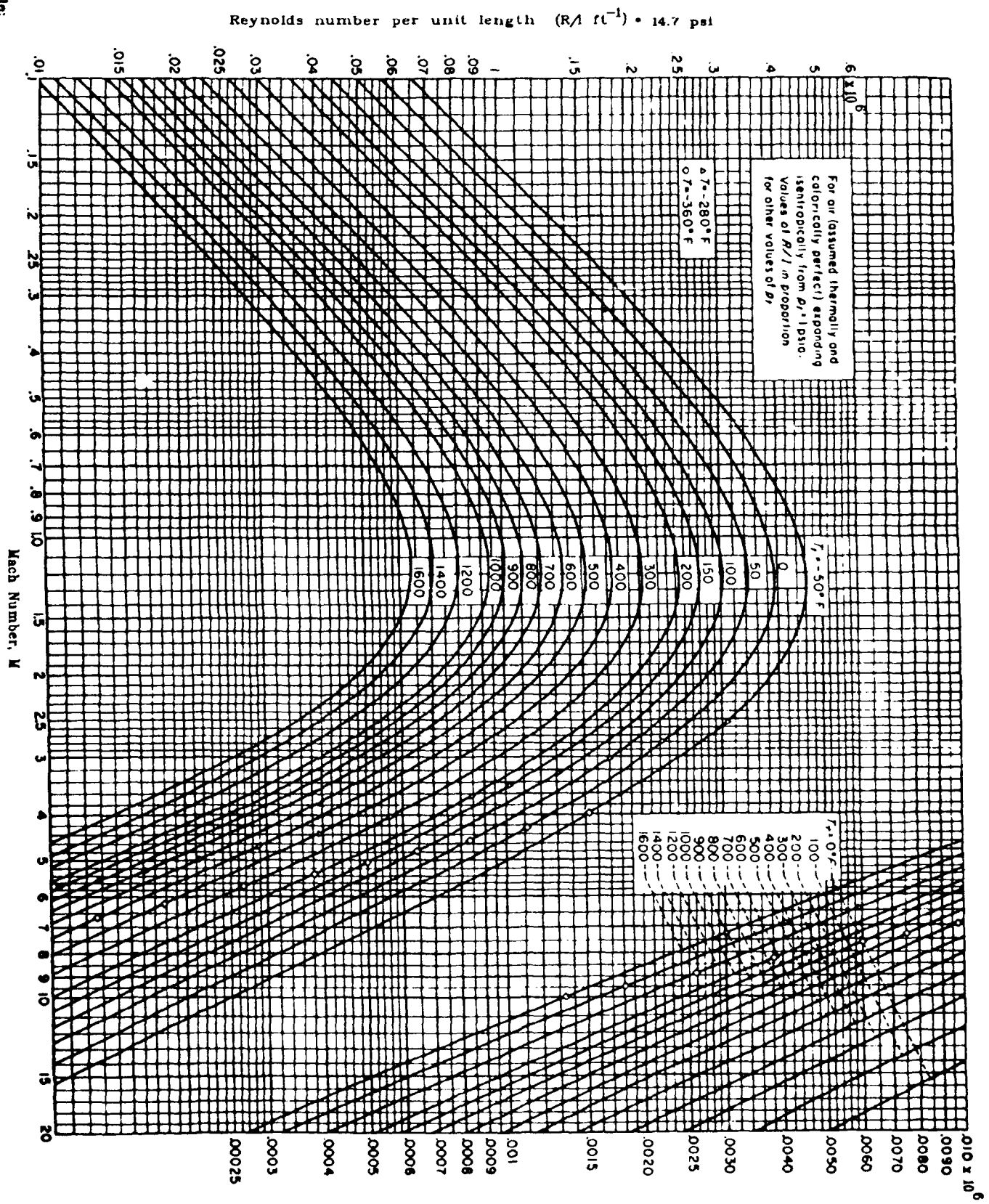


Figure A-2 Reynolds Number Determination (ref. 1)



Altitude ft	Temperature		Pressure		Density		Speed of Sound		Absolute Viscosity lbm/ft-s	Kinematic Viscosity ft <sup>2</sup> /sec
	F	R	psf	psi	lbm/ft <sup>3</sup>	ro/ro <sub>0</sub>	ft/sec	mi/hr		
0	58.7	518.4	2116.0	14.700	0.002378	1.0000	1107.64	760.9	3.725x10 <sup>-7</sup>	1.566x10 <sup>-4</sup>
2000	51.5	511.2	1968.0	13.672	0.002242	0.9428	1100.07	755.7	3.685	1.644
4000	44.4	504.1	1828.0	12.699	0.002112	0.8881	1092.36	750.4	3.644	1.725
6000	37.3	497.0	1696.0	11.782	0.001988	0.8358	1084.64	745.1	3.602	1.812
8000	30.2	489.9	1572.0	10.921	0.001869	0.7859	1076.78	739.7	3.561x10 <sup>-7</sup>	1.905x10 <sup>-4</sup>
10000	23.0	482.7	1455.0	10.108	0.001756	0.7384	1068.92	734.3	3.519	2.004
12000	15.9	475.6	1346.0	9.351	0.001648	0.6931	1060.91	728.8	3.476	2.109
14000	8.8	468.5	1243.0	8.635	0.001545	0.6499	1053.05	723.4	3.434	2.223
16000	1.6	461.3	1146.0	7.961	0.001448	0.6088	1046.21	718.7	3.391x10 <sup>-7</sup>	2.342x10 <sup>-4</sup>
18000	-5.5	454.2	1056.0	7.336	0.001355	0.5698	1036.75	712.2	3.348	2.471
20000	-12.6	447.1	972.1	6.753	0.001267	0.5327	1028.60	706.6	3.305	2.608
22000	-19.8	439.9	893.3	6.206	0.001183	0.4974	1020.59	701.1	3.261	2.756
24000	-26.9	432.8	819.8	5.695	0.001103	0.4640	1012.15	695.3	3.217x10 <sup>-7</sup>	2.916x10 <sup>-4</sup>
26000	-34.0	425.7	751.2	5.219	0.001028	0.4323	1003.71	689.5	3.173	3.086
28000	-41.2	418.5	687.4	4.775	0.000957	0.4023	995.26	683.7	3.128	3.268
30000	-48.3	411.4	628.0	4.363	0.000889	0.3740	986.82	677.9	3.083	3.468
32000	-55.4	404.3	572.9	3.980	0.000826	0.3472	978.23	672.0	3.038x10 <sup>-7</sup>	3.678x10 <sup>-4</sup>
34000	-62.5	397.2	521.7	3.624	0.000765	0.3218	969.50	666.0	2.992	3.911
36000	-67.3	392.4	474.4	3.296	0.000705	0.2963	963.67	662.0	2.962	4.204
38000	-67.3	392.4	431.1	2.995	0.000640	0.2692	963.67	662.0	2.962	4.625
40000	-67.3	392.4	391.9	2.723	0.000582	0.2448	963.67	662.0	2.962x10 <sup>-7</sup>	5.089x10 <sup>-4</sup>
42000	-67.3	392.4	356.2	2.475	0.000529	0.2225	963.67	662.0	2.962	5.599
44000	-67.3	392.4	323.7	2.249	0.000480	0.2021	963.67	662.0	2.962	6.161
46000	-67.3	392.4	294.2	2.044	0.000437	0.1838	963.67	662.0	2.962	6.778
48000	-67.3	392.4	267.4	1.858	0.000397	0.1670	963.67	662.0	2.962x10 <sup>-7</sup>	7.459x10 <sup>-4</sup>
50000	-67.3	392.4	243.1	1.689	0.000361	0.1518	963.67	662.0	2.962	8.206
52000	-67.3	392.4	220.9	1.535	0.000328	0.1379	963.67	662.0	2.962	9.028
54000	-67.3	392.4	200.8	1.395	0.000298	0.1254	963.67	662.0	2.962	9.933
56000	-67.3	392.4	182.5	1.268	0.000271	0.1140	963.67	662.0	2.962x10 <sup>-7</sup>	10.93x10 <sup>-4</sup>
58000	-67.3	392.4	165.9	1.153	0.000246	0.1036	963.67	662.0	2.962	12.02
60000	-67.3	392.4	150.8	1.048	0.000224	0.09415	963.67	662.0	2.962	13.23
62000	-67.3	392.4	137.1	0.952	0.000203	0.08557	963.67	662.0	2.962	14.56
64000	-67.3	392.4	124.6	0.866	0.000185	0.07777	963.67	662.0	2.962x10 <sup>-7</sup>	16.02x10 <sup>-4</sup>
65000	-67.3	392.4	118.7	0.825	0.000176	0.07414	963.67	662.0	2.962	16.8
70000	-67.3	392.4	93.53	0.650	0.000139	0.05839	963.67	662.0	2.962	21.33
75000	-67.3	392.4	73.66	0.512	0.000109	0.04599	963.67	662.0	2.962	27.09

Figure A-3 Standard Atmosphere

# EQUATIONS, TABLES, AND CHARTS FOR COMPRESSIBLE FLOW

## SUBSONIC FLOW

$\gamma = 1.4$

M	$\frac{p}{p_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\beta$	$\frac{A}{A_1}$	$\frac{V}{V_1}$	M	$\frac{p}{p_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\beta$	$\frac{A}{A_1}$	$\frac{V}{V_1}$	
0	1.0000	1.0000	1.0000	1.0000	0	0	0.50	0.8430	0.8552	0.9524	0.0690	0.1473	1.3398	0.53452
01	0.9998	0.9998	0.9998	0.9999	0.0000	0.0100	0.51	0.8374	0.8496	0.9508	0.0702	0.1525	1.3212	0.54489
02	0.9987	0.9986	0.9986	0.9996	0.0004	0.0200	0.52	0.8317	0.8439	0.9487	0.0714	0.1577	1.3024	0.55481
03	0.9969	0.9966	0.9966	0.9986	0.0014	0.0300	0.53	0.8259	0.8380	0.9466	0.0726	0.1629	1.2834	0.56431
04	0.9944	0.9939	0.9939	0.9972	0.0027	0.0400	0.54	0.8201	0.8321	0.9449	0.0738	0.1674	1.2642	0.57341
05	0.9913	0.9905	0.9905	0.9953	0.0047	0.0500	0.55	0.8142	0.8262	0.9430	0.0750	0.1724	1.2450	0.58206
06	0.9875	0.9863	0.9863	0.9929	0.0071	0.0600	0.56	0.8082	0.8203	0.9410	0.0762	0.1774	1.2263	0.59027
07	0.9830	0.9814	0.9814	0.9898	0.0103	0.0700	0.57	0.8022	0.8144	0.9390	0.0774	0.1825	1.2081	0.59805
08	0.9778	0.9758	0.9758	0.9862	0.0144	0.0800	0.58	0.7962	0.8085	0.9370	0.0786	0.1876	1.1904	0.60548
09	0.9719	0.9694	0.9694	0.9822	0.0194	0.0900	0.59	0.7901	0.8026	0.9349	0.0798	0.1925	1.1732	0.61257
10	0.9653	0.9623	0.9623	0.9777	0.0254	0.1000	0.60	0.7840	0.7966	0.9328	0.0810	0.1976	1.1564	0.61934
11	0.9580	0.9545	0.9545	0.9727	0.0324	0.1100	0.61	0.7778	0.7907	0.9307	0.0822	0.2027	1.1399	0.62579
12	0.9500	0.9460	0.9460	0.9672	0.0404	0.1200	0.62	0.7716	0.7848	0.9286	0.0834	0.2078	1.1237	0.63194
13	0.9413	0.9368	0.9368	0.9613	0.0494	0.1300	0.63	0.7654	0.7784	0.9265	0.0846	0.2127	1.1077	0.63780
14	0.9320	0.9270	0.9270	0.9550	0.0594	0.1400	0.64	0.7591	0.7721	0.9243	0.0858	0.2177	1.0919	0.64338
15	0.9221	0.9166	0.9166	0.9483	0.0704	0.1500	0.65	0.7528	0.7658	0.9221	0.0870	0.2227	1.0763	0.64877
16	0.9116	0.9056	0.9056	0.9412	0.0824	0.1600	0.66	0.7465	0.7595	0.9199	0.0882	0.2277	1.0609	0.65397
17	0.8995	0.8930	0.8930	0.9338	0.0954	0.1700	0.67	0.7401	0.7530	0.9176	0.0894	0.2326	1.0457	0.65897
18	0.8859	0.8788	0.8788	0.9261	0.1094	0.1800	0.68	0.7338	0.7468	0.9153	0.0906	0.2375	1.0307	0.66377
19	0.8710	0.8634	0.8634	0.9181	0.1244	0.1900	0.69	0.7274	0.7405	0.9131	0.0918	0.2424	1.0158	0.66837
20	0.8548	0.8467	0.8467	0.9098	0.1404	0.2000	0.70	0.7209	0.7340	0.9107	0.0930	0.2473	1.0011	0.67277
21	0.8374	0.8288	0.8288	0.9012	0.1574	0.2100	0.71	0.7145	0.7276	0.9084	0.0942	0.2521	0.9867	0.67697
22	0.8189	0.8098	0.8098	0.8923	0.1754	0.2200	0.72	0.7080	0.7211	0.9060	0.0954	0.2569	0.9725	0.68097
23	0.7994	0.7898	0.7898	0.8831	0.1944	0.2300	0.73	0.7016	0.7147	0.9037	0.0966	0.2617	0.9587	0.68477
24	0.7789	0.7688	0.7688	0.8736	0.2144	0.2400	0.74	0.6951	0.7082	0.9013	0.0978	0.2664	0.9453	0.68837
25	0.7574	0.7467	0.7467	0.8638	0.2354	0.2500	0.75	0.6886	0.7017	0.8990	0.0990	0.2711	0.9324	0.69177
26	0.7349	0.7237	0.7237	0.8537	0.2574	0.2600	0.76	0.6821	0.6952	0.8964	0.1002	0.2758	0.9200	0.69497
27	0.7114	0.6997	0.6997	0.8433	0.2804	0.2700	0.77	0.6756	0.6887	0.8939	0.1014	0.2805	0.9081	0.69797
28	0.6869	0.6747	0.6747	0.8326	0.3044	0.2800	0.78	0.6691	0.6822	0.8915	0.1026	0.2851	0.8967	0.70077
29	0.6614	0.6487	0.6487	0.8216	0.3294	0.2900	0.79	0.6625	0.6756	0.8890	0.1038	0.2899	0.8857	0.70337
30	0.6350	0.6218	0.6218	0.8103	0.3554	0.3000	0.80	0.6560	0.6691	0.8865	0.1050	0.2946	0.8751	0.70587
31	0.6077	0.5940	0.5940	0.7987	0.3824	0.3100	0.81	0.6495	0.6626	0.8840	0.1062	0.2993	0.8649	0.70817
32	0.5796	0.5654	0.5654	0.7868	0.4104	0.3200	0.82	0.6430	0.6561	0.8815	0.1074	0.3039	0.8551	0.71027
33	0.5507	0.5359	0.5359	0.7746	0.4394	0.3300	0.83	0.6365	0.6496	0.8790	0.1086	0.3085	0.8457	0.71217
34	0.5211	0.5058	0.5058	0.7621	0.4694	0.3400	0.84	0.6300	0.6431	0.8765	0.1098	0.3131	0.8367	0.71397
35	0.4908	0.4799	0.4799	0.7494	0.5004	0.3500	0.85	0.6235	0.6366	0.8740	0.1110	0.3177	0.8280	0.71567
36	0.4599	0.4485	0.4485	0.7364	0.5324	0.3600	0.86	0.6170	0.6301	0.8715	0.1122	0.3223	0.8196	0.71717
37	0.4284	0.4165	0.4165	0.7231	0.5654	0.3700	0.87	0.6105	0.6236	0.8690	0.1134	0.3269	0.8114	0.71847
38	0.3963	0.3839	0.3839	0.7096	0.5994	0.3800	0.88	0.6040	0.6171	0.8665	0.1146	0.3315	0.8034	0.71957
39	0.3637	0.3508	0.3508	0.6959	0.6344	0.3900	0.89	0.5975	0.6106	0.8640	0.1158	0.3361	0.7956	0.72047
40	0.3307	0.3173	0.3173	0.6822	0.6704	0.4000	0.90	0.5910	0.6037	0.8615	0.1170	0.3407	0.7878	0.72117
41	0.2973	0.2833	0.2833	0.6684	0.7074	0.4100	0.91	0.5845	0.5968	0.8590	0.1182	0.3453	0.7801	0.72167
42	0.2635	0.2489	0.2489	0.6545	0.7454	0.4200	0.92	0.5780	0.5903	0.8565	0.1194	0.3499	0.7725	0.72197
43	0.2293	0.2137	0.2137	0.6405	0.7844	0.4300	0.93	0.5715	0.5838	0.8540	0.1206	0.3545	0.7650	0.72207
44	0.1947	0.1785	0.1785	0.6264	0.8244	0.4400	0.94	0.5650	0.5773	0.8515	0.1218	0.3591	0.7575	0.72207
45	0.1598	0.1430	0.1430	0.6122	0.8654	0.4500	0.95	0.5585	0.5708	0.8490	0.1230	0.3637	0.7500	0.72197
46	0.1246	0.1075	0.1075	0.5979	0.9074	0.4600	0.96	0.5520	0.5643	0.8465	0.1242	0.3683	0.7425	0.72167
47	0.0892	0.0819	0.0819	0.5832	0.9504	0.4700	0.97	0.5455	0.5578	0.8440	0.1254	0.3729	0.7350	0.72117
48	0.0536	0.0460	0.0460	0.5684	0.9944	0.4800	0.98	0.5390	0.5513	0.8415	0.1266	0.3775	0.7275	0.72047
49	0.0179	0.0159	0.0159	0.5535	1.0394	0.4900	0.99	0.5325	0.5448	0.8390	0.1278	0.3821	0.7200	0.71957
1.00							1.00	0.5283	0.6339	0.8333	0.0000	0.6688	1.0000	1.00000

TABLE II.—SUPERSONIC FLOW

$\gamma = 1.4$

$M$ or $M_1$	$\frac{p}{p_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\beta$	$\frac{p}{p_0}$	$\frac{A}{A^*}$	$\frac{V}{a^*}$	$\nu$	$\mu$	$M_1$	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_3}{p_1}$	$\frac{p_4}{p_1}$
1.00	0.5283	0.6339	0.8333	0	0.7209	1.000	1.0000	0	90.0	1.000	1.000	1.000	1.000	1.000	0.5283
1.01	0.5271		0.8306	14.18	0.7288	1.000	1.00031		81.93	0.901	1.023	1.017	1.007	1.000	0.5271
1.02	0.5160	0.6234	0.8278	28.10	0.7358	1.000	1.00334	12.57	76.64	0.853	1.047	1.033	1.013	1.000	0.5160
1.03	0.5099	0.6191	0.8250	42.02	0.7409	1.001	1.00614	25.94	76.14	0.812	1.060	1.050	1.024	1.000	0.5099
1.04	0.5019	0.6120	0.8222	55.94	0.7455	1.001	1.03300	33.10	74.06	0.820	1.095	1.067	1.026	0.9999	0.5019
1.06	0.4970	0.6077	0.8180	70.77	0.7502	1.002	1.04114	48.74	72.25	0.831	1.120	1.084	1.033	0.9999	0.4970
1.07	0.4919	0.6034	0.8138	84.60	0.7549	1.004	1.04848	63.65	70.63	0.844	1.145	1.101	1.039	0.9999	0.4919
1.08	0.4860	0.6072	0.8100	98.40	0.7593	1.006	1.05731	78.63	69.16	0.850	1.169	1.094	1.046	0.9998	0.4860
1.09	0.4800	0.6020	0.8062	112.19	0.7639	1.005	1.06533	93.60	67.81	0.857	1.194	1.135	1.052	0.9994	0.4800
1.09	0.4742	0.5969	0.8026	126.00	0.7684	1.006	1.07331	108.55	66.55	0.865	1.219	1.152	1.059	0.9992	0.4742
1.10	0.4684	0.5917	0.8002	140.83	0.7727	1.008	1.08124	123.46	65.38	0.871	1.245	1.169	1.065	0.9989	0.4684
1.11	0.4626	0.5866	0.8023	155.68	0.7769	1.010	1.08913	138.34	64.26	0.880	1.271	1.186	1.071	0.9986	0.4626
1.12	0.4568	0.5814	0.7994	170.54	0.7811	1.011	1.09699	153.23	63.23	0.886	1.297	1.203	1.078	0.9982	0.4568
1.13	0.4511	0.5863	0.7966	185.42	0.7852	1.013	1.10479	168.14	62.25	0.892	1.323	1.221	1.084	0.9978	0.4511
1.14	0.4455	0.5813	0.7937	200.31	0.7892	1.015	1.11256	183.00	61.31	0.898	1.350	1.238	1.090	0.9973	0.4455
1.15	0.4398	0.5862	0.7907	215.20	0.7932	1.017	1.12029	207.81	60.41	0.897	1.376	1.255	1.097	0.9967	0.4398
1.16	0.4343	0.5811	0.7879	230.10	0.7970	1.026	1.12797	232.67	59.55	0.903	1.403	1.272	1.103	0.9961	0.4343
1.17	0.4287	0.5761	0.7851	245.00	0.8007	1.022	1.13561	257.50	58.69	0.912	1.430	1.289	1.109	0.9957	0.4287
1.18	0.4232	0.5710	0.7824	260.00	0.8044	1.025	1.14321	282.32	57.94	0.919	1.458	1.307	1.115	0.9952	0.4232
1.19	0.4178	0.5661	0.7797	275.00	0.8081	1.026	1.15077	307.14	57.18	0.925	1.485	1.324	1.122	0.9947	0.4178
1.20	0.4124	0.5611	0.7769	290.00	0.8117	1.030	1.15829	331.98	56.44	0.942	1.513	1.342	1.128	0.9938	0.4124
1.21	0.4070	0.5562	0.7745	305.00	0.8152	1.033	1.16575	356.80	55.74	0.949	1.541	1.359	1.134	0.9918	0.4104
1.22	0.4017	0.5513	0.7716	320.00	0.8187	1.037	1.17319	381.62	55.05	0.950	1.570	1.376	1.141	0.9907	0.4055
1.23	0.3964	0.5464	0.7687	335.00	0.8222	1.040	1.18057	406.44	54.39	0.951	1.598	1.394	1.147	0.9894	0.4005
1.24	0.3912	0.5415	0.7658	350.00	0.8257	1.043	1.18792	431.26	53.75	0.951	1.627	1.411	1.153	0.9884	0.3955

## REPORT 1135—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## SUPERSONIC FLOW—Continued

 $\gamma = 7/5$ 

$M$ $\frac{V}{a}$	$\frac{p}{p_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\beta$	$\frac{q}{p_1}$	$\frac{A}{A^*}$	$\frac{V}{a^*}$	$\frac{V}{a}$	$\mu$	$M$	$\frac{p}{p_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\frac{p_2}{p_1}$	$\frac{p_2}{p_1}$
1.26	.3861	.6067	.7610	.7500	.4221	1.047	1.16323	4.830	53.13	.8126	1.656	1.429	1.150	.0671	.3911
1.27	.3809	.6019	.7560	.7466	.4233	1.050	1.20249	5.081	52.53	.8071	1.666	1.446	1.166	.0687	.3866
1.28	.3758	.5971	.7518	.7380	.4244	1.054	1.24072	5.359	51.94	.8016	1.715	1.463	1.172	.0694	.3819
1.29	.3708	.5923	.7476	.7332	.4253	1.058	1.27895	5.627	51.34	.7961	1.745	1.481	1.178	.0697	.3774
1.30	.3658	.5875	.7433	.7283	.4262	1.062	1.31718	5.894	50.75	.7911	1.775	1.498	1.185	.0691	.3729
1.31	.3609	.5829	.7389	.7234	.4270	1.066	1.35541	6.170	50.16	.7860	1.805	1.515	1.191	.0704	.3685
1.32	.3560	.5782	.7345	.7185	.4277	1.071	1.39364	6.445	49.57	.7809	1.835	1.533	1.197	.0706	.3642
1.33	.3512	.5736	.7301	.7136	.4283	1.075	1.43187	6.721	48.98	.7758	1.866	1.551	1.204	.0708	.3599
1.34	.3464	.5690	.7257	.7087	.4289	1.080	1.47010	7.000	48.39	.7707	1.897	1.569	1.210	.0718	.3557
1.35	.3417	.5644	.7213	.7038	.4294	1.084	1.50833	7.280	47.80	.7656	1.928	1.585	1.216	.0718	.3516
1.36	.3370	.5598	.7169	.6989	.4299	1.089	1.54656	7.561	47.21	.7605	1.959	1.603	1.223	.0697	.3475
1.37	.3323	.5552	.7125	.6940	.4303	1.094	1.58479	7.844	46.62	.7554	1.991	1.620	1.229	.0676	.3435
1.38	.3277	.5506	.7081	.6891	.4308	1.099	1.62302	8.128	46.03	.7503	2.023	1.638	1.235	.0655	.3395
1.39	.3232	.5460	.7037	.6842	.4312	1.104	1.66125	8.413	45.44	.7452	2.055	1.655	1.242	.0634	.3355
1.40	.3187	.5414	.7000	.6793	.4316	1.109	1.70000	8.699	44.85	.7401	2.087	1.672	1.248	.0607	.3317
1.41	.3142	.5368	.6962	.6744	.4321	1.115	1.73875	8.987	44.26	.7350	2.120	1.690	1.255	.0582	.3280
1.42	.3098	.5322	.6924	.6695	.4325	1.120	1.77750	9.276	43.67	.7300	2.153	1.707	1.261	.0557	.3242
1.43	.3055	.5277	.6886	.6646	.4329	1.126	1.81625	9.565	43.08	.7250	2.186	1.724	1.268	.0531	.3205
1.44	.3012	.5232	.6848	.6597	.4333	1.132	1.85500	9.855	42.49	.7200	2.219	1.742	1.274	.0504	.3169
1.45	.2969	.5187	.6810	.6548	.4337	1.138	1.89375	10.146	41.90	.7150	2.253	1.759	1.281	.0476	.3134
1.46	.2927	.5142	.6772	.6499	.4341	1.144	1.93250	10.438	41.31	.7100	2.286	1.776	1.287	.0448	.3099
1.47	.2885	.5097	.6734	.6450	.4345	1.150	1.97125	10.731	40.72	.7050	2.320	1.793	1.294	.0420	.3065
1.48	.2843	.5052	.6696	.6401	.4349	1.156	2.01000	11.024	40.13	.7000	2.354	1.811	1.301	.0392	.3031
1.49	.2801	.5007	.6658	.6352	.4353	1.162	2.04875	11.317	39.54	.6950	2.388	1.828	1.307	.0364	.2997
1.50	.2759	.4962	.6620	.6303	.4357	1.168	2.08750	11.611	38.95	.6900	2.423	1.845	1.314	.0336	.2962
1.51	.2717	.4917	.6582	.6254	.4361	1.174	2.12625	11.905	38.36	.6850	2.458	1.862	1.320	.0308	.2928
1.52	.2675	.4872	.6544	.6205	.4365	1.180	2.16500	12.200	37.77	.6800	2.493	1.879	1.327	.0280	.2894
1.53	.2633	.4827	.6506	.6156	.4369	1.186	2.20375	12.495	37.18	.6750	2.528	1.896	1.334	.0252	.2860
1.54	.2591	.4782	.6468	.6107	.4373	1.192	2.24250	12.790	36.59	.6700	2.563	1.913	1.340	.0224	.2826
1.55	.2549	.4737	.6430	.6058	.4377	1.198	2.28125	13.085	36.00	.6650	2.598	1.930	1.347	.0196	.2792
1.56	.2507	.4692	.6392	.6009	.4381	1.204	2.32000	13.380	35.41	.6600	2.633	1.947	1.354	.0168	.2758
1.57	.2465	.4647	.6354	.5960	.4385	1.210	2.35875	13.675	34.82	.6550	2.668	1.964	1.361	.0140	.2724
1.58	.2423	.4602	.6316	.5911	.4389	1.216	2.39750	13.970	34.23	.6500	2.703	1.981	1.367	.0112	.2690
1.59	.2381	.4557	.6278	.5862	.4393	1.222	2.43625	14.265	33.64	.6450	2.738	1.998	1.374	.0084	.2656
1.60	.2339	.4512	.6240	.5813	.4397	1.228	2.47500	14.560	33.05	.6400	2.773	2.015	1.381	.0056	.2622
1.61	.2297	.4467	.6202	.5764	.4401	1.234	2.51375	14.855	32.46	.6350	2.808	2.032	1.388	.0028	.2588
1.62	.2255	.4422	.6164	.5715	.4405	1.240	2.55250	15.150	31.87	.6300	2.843	2.049	1.395	.0000	.2554
1.63	.2213	.4377	.6126	.5666	.4409	1.246	2.59125	15.445	31.28	.6250	2.878	2.066	1.402	.0000	.2520
1.64	.2171	.4332	.6088	.5617	.4413	1.252	2.63000	15.740	30.69	.6200	2.913	2.083	1.409	.0000	.2486
1.65	.2129	.4287	.6050	.5568	.4417	1.258	2.66875	16.035	30.10	.6150	2.948	2.100	1.416	.0000	.2452
1.66	.2087	.4242	.6012	.5519	.4421	1.264	2.70750	16.330	29.51	.6100	2.983	2.117	1.423	.0000	.2418
1.67	.2045	.4197	.5974	.5470	.4425	1.270	2.74625	16.625	28.92	.6050	3.018	2.134	1.430	.0000	.2384
1.68	.2003	.4152	.5936	.5421	.4429	1.276	2.78500	16.920	28.33	.6000	3.053	2.151	1.437	.0000	.2350
1.69	.1961	.4107	.5898	.5372	.4433	1.282	2.82375	17.215	27.74	.5950	3.088	2.168	1.444	.0000	.2316
1.70	.1919	.4062	.5860	.5323	.4437	1.288	2.86250	17.510	27.15	.5900	3.123	2.185	1.451	.0000	.2282
1.71	.1877	.4017	.5822	.5274	.4441	1.294	2.90125	17.805	26.56	.5850	3.158	2.202	1.458	.0000	.2248
1.72	.1835	.3972	.5784	.5225	.4445	1.300	2.94000	18.100	25.97	.5800	3.193	2.219	1.465	.0000	.2214
1.73	.1793	.3927	.5746	.5176	.4449	1.306	2.97875	18.395	25.38	.5750	3.228	2.236	1.472	.0000	.2180
1.74	.1751	.3882	.5708	.5127	.4453	1.312	3.01750	18.690	24.79	.5700	3.263	2.253	1.479	.0000	.2146
1.75	.1709	.3837	.5670	.5078	.4457	1.318	3.05625	18.985	24.20	.5650	3.298	2.270	1.486	.0000	.2112
1.76	.1667	.3792	.5632	.5029	.4461	1.324	3.09500	19.280	23.61	.5600	3.333	2.287	1.493	.0000	.2078
1.77	.1625	.3747	.5594	.4980	.4465	1.330	3.13375	19.575	23.02	.5550	3.368	2.304	1.500	.0000	.2044
1.78	.1583	.3702	.5556	.4931	.4469	1.336	3.17250	19.870	22.43	.5500	3.403	2.321	1.507	.0000	.2010
1.79	.1541	.3657	.5518	.4882	.4473	1.342	3.21125	20.165	21.84	.5450	3.438	2.338	1.514	.0000	.1976
1.80	.1499	.3612	.5480	.4833	.4477	1.348	3.25000	20.460	21.25	.5400	3.473	2.355	1.521	.0000	.1942
1.81	.1457	.3567	.5442	.4784	.4481	1.354	3.28875	20.755	20.66	.5350	3.508	2.372	1.528	.0000	.1908
1.82	.1415	.3522	.5404	.4735	.4485	1.360	3.32750	21.050	20.07	.5300	3.543	2.389	1.535	.0000	.1874
1.83	.1373	.3477	.5366	.4686	.4489	1.366	3.36625	21.345	19.48	.5250	3.578	2.406	1.542	.0000	.1840
1.84	.1331	.3432	.5328	.4637	.4493	1.372	3.40500	21.640	18.89	.5200	3.613	2.423	1.549	.0000	.1806
1.85	.1289	.3387	.5290	.4588	.4497	1.378	3.44375	21.935	18.30	.5150	3.648	2.440	1.556	.0000	.1772
1.86	.1247	.3342	.5252	.4539	.4501	1.384	3.48250	22.230	17.71	.5100	3.683	2.457	1.563	.0000	.1738
1.87	.1205	.3297	.5214	.4490	.4505	1.390	3.52125	22.525	17.12	.5050	3.718	2.474	1.570	.0000	.1704
1.88	.1163	.3252	.5176	.4441	.4509	1.396	3.56000	22.820	16.53	.5000	3.753	2.491	1.577	.0000	.1670
1.89	.1121	.3207	.5138	.4392	.4513	1.402	3.59875	23.115	15.94	.4950	3.788	2.508	1.584	.0000	.1636
1.90	.1079	.3162	.5100	.4343	.4517	1.408	3.63750	23.410	15.35	.4900	3.823	2.525	1.591	.0000	.1602
1.91	.1037	.3117	.5062	.4294	.4521	1.414	3.67625	23.705	14.76	.4850	3.858	2.542	1.598	.0000	.1568
1.92	.1000	.3072	.5024	.4245	.4525	1.420	3.71500	24.000	14.17	.4800	3.893	2.559	1.605	.0000	.1534
1.93	.0962	.3027	.4986	.4196	.4529	1.426	3.75375	24.295	13.58	.4750	3.928	2.576	1.612	.0000	.1500
1.94	.0924	.2982	.4948	.4147	.4533	1.432	3.79250	24.590	12.99	.4700	3.963	2.593	1.619	.0000	.1466
1.95	.0886	.2937	.4910	.4098	.4537	1.438	3.83125	24.885	12.40	.4650	3.998	2.610	1.626	.0000	.1432
1.96	.0848	.2892	.4872	.4049	.4541	1.444	3.87000	25.180	11.81	.4600	4.033	2.627	1.633	.0000	.1398
1.97	.0810	.2847	.4834	.3999	.4545	1.450	3.90875	25.475	11.22	.4550	4.068	2.644	1.640	.0000	.1364
1.98	.0772	.2802	.4796	.3950	.4549	1.456	3.94750	25.770	10.63	.4500	4.103	2.661	1.647	.0000	.1330
1.99	.0734	.2757	.4758	.3901	.4553	1.462	3.98625	26.065	10.04	.4450	4.138	2.678	1.654	.0000	.1296
2.00	.0696	.2712	.4720	.3852	.4557	1.468	4.02500	26.360	9.45	.4400	4.173	2.695	1.661	.0000	.1262
2.01	.0658	.2667	.4682	.3803	.4561	1.474									

## EQUATIONS, TABLES, AND CHARTS FOR COMPRESSIBLE FLOW

## SUPersonic FLOW

 $\gamma = 1.4$ 

$M_1$	$\frac{P}{P_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\beta$	$\frac{q}{P_1}$	$\frac{A}{A_1}$	$\frac{V}{c}$	$V$	$\mu$	$M_1$	$\frac{P}{P_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\frac{P_2}{P_1}$	$\frac{P_2}{P_1}$
2.15	.1011	.1946	.5196	1.903	.3272	1.919	1.06774	30.425	27.72	.6540	5.226	2.882	1.813	.6511	.1553
2.16	.9966	.1925	.5173	1.915	.3232	1.935	1.07183	30.699	27.58	.5525	5.277	2.806	1.822	.6464	.1540
2.17	.9922	.1905	.5150	1.928	.3193	1.953	1.07599	30.951	27.44	.5511	5.327	2.910	1.831	.6419	.1527
2.18	.9879	.1882	.5127	1.937	.3150	1.970	1.08022	31.212	27.30	.5498	5.378	2.924	1.839	.6373	.1514
2.19	.9836	.1861	.5104	1.948	.3109	1.987	1.08453	31.473	27.17	.5484	5.429	2.938	1.848	.6327	.1502
2.20	.9793	.1841	.5081	1.960	.3069	2.005	1.08891	31.732	27.04	.5471	5.480	2.951	1.857	.6281	.1489
2.21	.9750	.1820	.5058	1.971	.3028	2.023	1.09337	31.991	26.90	.5457	5.531	2.965	1.866	.6236	.1476
2.22	.9707	.1800	.5035	1.982	.3027	2.041	1.09789	32.250	26.77	.5444	5.583	2.978	1.875	.6191	.1464
2.23	.9663	.1780	.5014	1.993	.3005	2.059	1.10247	32.507	26.64	.5431	5.636	2.992	1.883	.6145	.1452
2.24	.9620	.1760	.4991	2.004	.3065	2.078	1.10713	32.763	26.51	.5418	5.687	3.005	1.892	.6100	.1440
2.25	.9577	.1740	.4969	2.016	.3065	2.096	1.11187	33.018	26.39	.5405	5.740	3.019	1.901	.6055	.1428
2.26	.9534	.1721	.4947	2.027	.3044	2.115	1.11669	33.273	26.26	.5393	5.792	3.032	1.910	.6011	.1417
2.27	.9492	.1702	.4925	2.038	.3023	2.134	1.12150	33.527	26.14	.5381	5.845	3.045	1.919	.5966	.1405
2.28	.9450	.1683	.4903	2.049	.3003	2.154	1.12639	33.780	26.01	.5368	5.898	3.058	1.928	.5921	.1393
2.29	.9407	.1664	.4881	2.060	.2982	2.173	1.13127	34.032	25.89	.5356	5.951	3.071	1.938	.5877	.1382
2.30	.9365	.1645	.4859	2.071	.2961	2.193	1.13613	34.283	25.77	.5344	6.005	3.085	1.947	.5833	.1371
2.31	.9322	.1626	.4837	2.082	.2941	2.213	1.14100	34.533	25.65	.5332	6.059	3.098	1.956	.5789	.1360
2.32	.9280	.1607	.4815	2.093	.2920	2.233	1.14587	34.783	25.53	.5321	6.113	3.110	1.965	.5745	.1349
2.33	.9237	.1588	.4794	2.104	.2900	2.254	1.15073	35.031	25.42	.5309	6.167	3.123	1.974	.5702	.1338
2.34	.9195	.1569	.4773	2.116	.2879	2.274	1.15559	35.279	25.30	.5297	6.222	3.136	1.984	.5658	.1327
2.35	.9153	.1550	.4752	2.127	.2859	2.295	1.16045	35.526	25.18	.5285	6.276	3.149	1.993	.5615	.1317
2.36	.9111	.1531	.4731	2.138	.2839	2.316	1.16531	35.771	25.07	.5273	6.331	3.162	2.002	.5572	.1306
2.37	.9069	.1512	.4710	2.149	.2818	2.338	1.17017	36.016	24.96	.5261	6.386	3.175	2.012	.5529	.1296
2.38	.9027	.1493	.4689	2.160	.2798	2.359	1.17503	36.261	24.85	.5250	6.442	3.187	2.021	.5486	.1286
2.39	.8985	.1474	.4668	2.171	.2778	2.381	1.17989	36.504	24.73	.5242	6.497	3.199	2.031	.5444	.1276
2.40	.8943	.1455	.4647	2.182	.2758	2.403	1.18475	36.748	24.62	.5231	6.553	3.212	2.040	.5401	.1266
2.41	.8901	.1436	.4626	2.193	.2738	2.425	1.18961	36.992	24.52	.5221	6.609	3.224	2.050	.5359	.1257
2.42	.8859	.1417	.4605	2.204	.2718	2.448	1.19447	37.235	24.41	.5210	6.666	3.237	2.059	.5317	.1247
2.43	.8817	.1398	.4584	2.215	.2698	2.471	1.19933	37.478	24.30	.5200	6.722	3.249	2.069	.5275	.1237
2.44	.8775	.1379	.4563	2.226	.2678	2.494	1.20419	37.720	24.19	.5189	6.779	3.261	2.079	.5234	.1228
2.45	.8733	.1360	.4542	2.237	.2658	2.517	1.20905	37.963	24.09	.5179	6.835	3.273	2.088	.5193	.1218
2.46	.8691	.1341	.4521	2.248	.2639	2.540	1.21391	38.205	23.99	.5168	6.894	3.285	2.098	.5152	.1209
2.47	.8649	.1322	.4500	2.259	.2619	2.564	1.21877	38.447	23.88	.5158	6.951	3.298	2.108	.5111	.1200
2.48	.8607	.1303	.4479	2.269	.2599	2.588	1.22363	38.689	23.78	.5149	7.009	3.310	2.118	.5071	.1191
2.49	.8565	.1284	.4458	2.280	.2580	2.612	1.22849	38.930	23.68	.5140	7.067	3.321	2.128	.5030	.1182
2.50	.8523	.1265	.4437	2.291	.2561	2.637	1.23335	39.171	23.58	.5130	7.125	3.333	2.138	.4990	.1173
2.51	.8481	.1246	.4416	2.302	.2541	2.661	1.23821	39.412	23.48	.5120	7.183	3.345	2.147	.4950	.1164
2.52	.8439	.1227	.4395	2.313	.2522	2.686	1.24307	39.653	23.38	.5111	7.242	3.357	2.157	.4911	.1155
2.53	.8397	.1208	.4374	2.324	.2503	2.712	1.24793	39.894	23.28	.5102	7.301	3.369	2.167	.4871	.1147
2.54	.8355	.1189	.4353	2.335	.2484	2.737	1.25279	40.135	23.18	.5092	7.360	3.380	2.177	.4832	.1138
2.55	.8313	.1170	.4332	2.346	.2465	2.762	1.25765	40.376	23.09	.5083	7.419	3.392	2.187	.4793	.1130
2.56	.8271	.1151	.4311	2.357	.2446	2.789	1.26251	40.617	22.99	.5074	7.479	3.403	2.198	.4754	.1122
2.57	.8229	.1132	.4290	2.368	.2427	2.815	1.26737	40.858	22.91	.5065	7.539	3.415	2.208	.4715	.1113
2.58	.8187	.1113	.4269	2.379	.2408	2.842	1.27223	41.099	22.81	.5056	7.599	3.426	2.219	.4677	.1105
2.59	.8145	.1094	.4248	2.389	.2389	2.869	1.27709	41.340	22.71	.5047	7.659	3.438	2.229	.4639	.1097
2.60	.8103	.1075	.4227	2.400	.2370	2.896	1.28195	41.581	22.62	.5039	7.720	3.449	2.238	.4601	.1090
2.61	.8061	.1056	.4206	2.411	.2351	2.923	1.28681	41.822	22.53	.5030	7.781	3.460	2.249	.4564	.1081
2.62	.8019	.1037	.4185	2.422	.2332	2.951	1.29167	42.063	22.44	.5022	7.842	3.471	2.259	.4526	.1074
2.63	.7977	.1018	.4164	2.433	.2313	2.979	1.29653	42.304	22.35	.5013	7.903	3.483	2.269	.4489	.1066
2.64	.7935	.0999	.4143	2.444	.2294	3.007	1.30139	42.545	22.26	.5005	7.965	3.494	2.280	.4452	.1058
2.65	.7893	.0980	.4122	2.455	.2275	3.036	1.30625	42.786	22.17	.4996	8.026	3.505	2.290	.4415	.1051
2.66	.7851	.0961	.4101	2.466	.2256	3.065	1.31111	43.027	22.08	.4988	8.088	3.516	2.301	.4379	.1043
2.67	.7809	.0942	.4080	2.477	.2237	3.094	1.31597	43.268	22.00	.4980	8.150	3.527	2.311	.4343	.1036
2.68	.7767	.0923	.4059	2.488	.2218	3.123	1.32083	43.509	21.91	.4972	8.213	3.537	2.322	.4307	.1028
2.69	.7725	.0904	.4038	2.499	.2199	3.153	1.32569	43.750	21.82	.4964	8.275	3.548	2.332	.4271	.1021
2.70	.7683	.0885	.4017	2.509	.2180	3.183	1.33055	44.000	21.74	.4956	8.338	3.559	2.343	.4236	.1014
2.71	.7641	.0866	.4000	2.519	.2161	3.213	1.33541	44.241	21.65	.4949	8.401	3.570	2.354	.4201	.1007
2.72	.7599	.0847	.3979	2.530	.2142	3.244	1.34027	44.482	21.57	.4941	8.465	3.580	2.364	.4166	.1000
2.73	.7557	.0828	.3958	2.540	.2123	3.275	1.34513	44.723	21.49	.4933	8.528	3.591	2.375	.4131	.9993
2.74	.7515	.0809	.3937	2.551	.2104	3.306	1.35000	44.964	21.41	.4926	8.592	3.601	2.386	.4097	.9986
2.75	.7473	.0790	.3916	2.562	.2085	3.338	1.35486	45.205	21.32	.4918	8.656	3.612	2.397	.4062	.9979
2.76	.7431	.0771	.3895	2.572	.2066	3.370	1.35972	45.446	21.24	.4911	8.721	3.622	2.407	.4028	.9972
2.77	.7389	.0752	.3874	2.583	.2047	3.402	1.36458	45.687	21.16	.4903	8.785	3.633	2.418	.3994	.9965
2.78	.7347	.0733	.3853	2.594	.2028	3.434	1.36944	45.928	21.08	.4896	8.850	3.643	2.429	.3961	.9958
2.79	.7305	.0714	.3832	2.605	.2009	3.467	1.37430	46.169	21.00	.4889	8.915	3.653	2.440	.3928	.9951
2.80	.7263	.0695	.3811	2.616	.2022	3.500	1.37916	46.410	20.92	.4882	8.980	3.664	2.451	.3895	.9944
2.81	.7221	.0676	.3790	2.627	.2003	3.534	1.38402	46.651	20.85	.4875	9.045	3.674	2.462	.3862	.9937
2.82	.7179	.0657	.3769	2.638	.2022	3.567	1.38888	46.892	20.77	.4868	9.111	3.684	2.473	.3829	.9930
2.83	.7137	.0638	.3748	2.649	.2003	3.601	1.39374	47.133	20.69	.4861	9.177	3.694	2.484	.3797	.9923
2.84	.7095	.0619	.3727	2.660	.2022	3.636	1.39860	47.374	20.62	.4854	9.243	3.704	2.496	.3765	.9916
2.85	.7053	.0600	.3706	2.671	.2003	3.671	1.40346	47.615	20.54	.4847	9.310	3.714	2.507	.3733	.9909
2.86	.7011	.0581	.3685	2.682	.2022	3.706	1.40832	47.856	20.47	.4840	9.376	3.724	2.518	.3701	.9902
2.87	.6969	.0562	.3664	2.693	.2003	3.741	1.41318	48.097	20.39	.4833	9.443	3.734	2.529	.3669	.9895
2.88	.6927	.0543	.3643	2.704	.2022	3.777	1.41804	48.338	20.32	.4827	9.510	3.743	2.540	.3637	.9888
2.89	.6885	.0524	.3622	2.715	.2003	3.813	1.42290	48.579	20.24	.4820	9.577	3.753	2.552	.3605	.9881

## REPORT 1135—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## SUPersonic FLOW

$\gamma = 7/5$

$M$ $M_1$	$\frac{p}{p_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\beta$	$\frac{e}{p_1}$	$\frac{A}{A_1}$	$\frac{V}{c}$	$V$	$\mu$	$M_1$	$\frac{p}{p_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\frac{p_2}{p_1}$	$\frac{p_2}{p_1}$
3.05	2536	7226	3498	2.881	1445	4.441	1.97547	50.713	19.14	4.723	10.69	3.902	2.738	3145	9072
3.06	2489	7149	3481	2.892	1431	4.483	1.97772	50.902	19.07	4.717	10.76	3.911	2.750	3118	7082
3.07	2452	7074	3466	2.903	1418	4.526	1.97997	51.080	19.01	4.712	10.83	3.920	2.762	3091	7032
3.08	2416	6999	3452	2.913	1404	4.570	1.98219	51.277	18.95	4.706	10.90	3.929	2.774	3065	7082
3.09	2380	6925	3437	2.924	1391	4.613	1.98441	51.464	18.88	4.701	10.97	3.938	2.786	3038	7032
3.10	2345	6852	3422	2.934	1377	4.657	1.98661	51.650	18.82	4.695	11.05	3.947	2.799	3012	7782
3.11	2310	6779	3407	2.945	1364	4.702	1.98879	51.835	18.76	4.690	11.12	3.955	2.811	2986	7732
3.12	2276	6708	3393	2.955	1351	4.747	1.99097	52.020	18.69	4.685	11.19	3.964	2.823	2960	7682
3.13	2243	6637	3379	2.966	1338	4.792	1.99313	52.203	18.63	4.679	11.26	3.973	2.835	2934	7632
3.14	2210	6566	3365	2.977	1325	4.838	1.99527	52.386	18.57	4.674	11.34	3.981	2.848	2910	7582
3.15	2177	6496	3351	2.987	1312	4.884	1.99740	52.569	18.51	4.669	11.41	3.990	2.860	2885	7532
3.16	2145	6426	3337	2.998	1300	4.930	1.99952	52.751	18.45	4.664	11.48	3.998	2.872	2860	7482
3.17	2114	6356	3323	3.008	1287	4.977	2.00162	52.932	18.39	4.659	11.56	4.006	2.885	2835	7432
3.18	2083	6286	3309	3.019	1275	5.022	2.00372	53.112	18.33	4.654	11.63	4.015	2.897	2811	7382
3.19	2053	6217	3295	3.029	1262	5.073	2.00579	53.292	18.27	4.649	11.71	4.023	2.909	2786	7332
3.20	2023	6148	3281	3.040	1250	5.121	2.00786	53.470	18.21	4.643	11.78	4.031	2.922	2762	7282
3.21	1993	6079	3267	3.050	1238	5.170	2.00991	53.648	18.15	4.638	11.85	4.040	2.935	2738	7232
3.22	1964	6011	3253	3.061	1226	5.219	2.01195	53.826	18.09	4.633	11.93	4.048	2.947	2715	7182
3.23	1936	5943	3239	3.071	1214	5.268	2.01398	54.003	18.03	4.628	12.01	4.056	2.960	2691	7132
3.24	1908	5875	3225	3.082	1202	5.319	2.01599	54.179	17.98	4.624	12.08	4.064	2.972	2668	7082
3.25	1880	5807	3211	3.092	1190	5.368	2.01799	54.355	17.92	4.619	12.16	4.072	2.985	2645	7032
3.26	1853	5740	3197	3.103	1178	5.418	2.01996	54.530	17.86	4.614	12.23	4.080	2.998	2622	6982
3.27	1826	5673	3183	3.113	1166	5.472	2.02190	54.703	17.81	4.610	12.31	4.088	3.011	2600	6932
3.28	1799	5607	3169	3.124	1154	5.523	2.02382	54.877	17.75	4.605	12.38	4.096	3.023	2577	6882
3.29	1773	5541	3155	3.134	1144	5.576	2.02582	55.050	17.70	4.600	12.46	4.104	3.036	2555	6832
3.30	1748	5475	3141	3.145	1132	5.629	2.02781	55.222	17.64	4.596	12.54	4.112	3.049	2533	6782
3.31	1722	5409	3127	3.155	1120	5.682	2.02978	55.393	17.58	4.591	12.62	4.120	3.062	2511	6732
3.32	1698	5343	3113	3.166	1108	5.736	2.03165	55.564	17.53	4.587	12.69	4.128	3.075	2489	6682
3.33	1673	5278	3099	3.176	1096	5.790	2.03356	55.734	17.48	4.582	12.77	4.135	3.088	2468	6632
3.34	1649	5212	3085	3.187	1084	5.845	2.03545	55.904	17.42	4.578	12.85	4.143	3.101	2446	6582
3.35	1625	5147	3071	3.197	1072	5.900	2.03733	56.073	17.37	4.573	12.93	4.151	3.114	2425	6532
3.36	1602	5081	3057	3.208	1060	5.956	2.03920	56.241	17.31	4.569	13.01	4.158	3.127	2404	6482
3.37	1579	5016	3043	3.218	1048	6.012	2.04106	56.409	17.26	4.565	13.08	4.166	3.141	2383	6432
3.38	1557	4951	3029	3.229	1036	6.069	2.04291	56.576	17.21	4.560	13.16	4.173	3.154	2361	6382
3.39	1534	4886	3015	3.239	1024	6.126	2.04474	56.742	17.16	4.556	13.24	4.181	3.167	2340	6332
3.40	1512	4821	3001	3.250	1012	6.184	2.04656	56.907	17.10	4.552	13.32	4.188	3.180	2319	6282
3.41	1491	4756	2987	3.260	1000	6.242	2.04837	57.073	17.05	4.548	13.40	4.196	3.194	2298	6232
3.42	1470	4691	2973	3.271	988	6.301	2.05017	57.237	17.00	4.544	13.48	4.203	3.207	2277	6182
3.43	1449	4626	2959	3.281	976	6.360	2.05196	57.401	16.95	4.540	13.56	4.211	3.220	2256	6132
3.44	1428	4561	2945	3.291	964	6.420	2.05374	57.564	16.90	4.535	13.64	4.218	3.234	2235	6082
3.45	1408	4496	2931	3.302	952	6.480	2.05551	57.726	16.85	4.531	13.72	4.225	3.247	2214	6032
3.46	1388	4431	2917	3.312	940	6.541	2.05727	57.888	16.80	4.527	13.80	4.232	3.261	2193	5982
3.47	1368	4366	2903	3.323	928	6.602	2.05901	58.050	16.75	4.523	13.88	4.240	3.274	2172	5932
3.48	1348	4301	2889	3.333	916	6.664	2.06074	58.210	16.70	4.519	13.96	4.247	3.288	2151	5882
3.49	1328	4236	2875	3.344	904	6.727	2.06247	58.370	16.65	4.515	14.04	4.254	3.301	2130	5832
3.50	1311	4171	2861	3.354	892	6.790	2.06419	58.530	16.60	4.512	14.11	4.261	3.315	2109	5782
3.51	1290	4106	2847	3.365	880	6.853	2.06590	58.689	16.55	4.508	14.19	4.268	3.328	2088	5732
3.52	1274	4041	2833	3.375	868	6.917	2.06760	58.847	16.51	4.504	14.27	4.275	3.341	2067	5682
3.53	1258	3976	2819	3.385	856	6.982	2.06927	59.004	16.46	4.500	14.35	4.282	3.354	2046	5632
3.54	1243	3911	2805	3.396	844	7.047	2.07091	59.162	16.41	4.496	14.43	4.289	3.370	2025	5582
3.55	1221	3846	2791	3.406	832	7.111	2.07253	59.318	16.36	4.492	14.51	4.296	3.384	2004	5532
3.56	1201	3781	2777	3.417	820	7.176	2.07416	59.474	16.31	4.489	14.59	4.303	3.398	1983	5482
3.57	1180	3716	2763	3.427	808	7.241	2.07578	59.629	16.27	4.485	14.67	4.309	3.412	1962	5432
3.58	1161	3651	2749	3.437	796	7.307	2.07739	59.783	16.22	4.481	14.75	4.316	3.426	1941	5382
3.59	1141	3586	2735	3.448	784	7.372	2.07900	59.938	16.17	4.477	14.83	4.323	3.440	1920	5332
3.60	1121	3521	2721	3.458	772	7.437	2.08060	60.091	16.13	4.474	14.91	4.330	3.454	1899	5282
3.61	1102	3456	2707	3.469	760	7.502	2.08219	60.244	16.08	4.471	15.00	4.336	3.468	1878	5232
3.62	1083	3391	2693	3.479	748	7.568	2.08377	60.397	16.04	4.467	15.08	4.343	3.482	1857	5182
3.63	1064	3326	2679	3.490	736	7.633	2.08535	60.549	16.00	4.463	15.16	4.350	3.496	1836	5132
3.64	1045	3261	2665	3.500	724	7.699	2.08691	60.700	15.95	4.460	15.24	4.356	3.510	1815	5082
3.65	1026	3196	2651	3.510	712	7.764	2.08847	60.851	15.90	4.456	15.32	4.363	3.523	1794	5032
3.66	1007	3131	2637	3.521	700	7.830	2.08999	61.000	15.86	4.451	15.40	4.369	3.537	1773	4982
3.67	1002	3116	2623	3.531	688	7.897	2.09149	61.149	15.81	4.447	15.48	4.375	3.551	1752	4932
3.68	1018	3101	2609	3.542	676	7.964	2.09296	61.298	15.77	4.443	15.56	4.382	3.564	1731	4882
3.69	1004	3086	2595	3.552	664	8.031	2.09441	61.447	15.72	4.440	15.64	4.388	3.578	1710	4832
3.70	983	3021	2581	3.562	652	8.098	2.09586	61.595	15.68	4.436	15.72	4.395	3.591	1689	4782
3.71	977	3006	2567	3.573	640	8.165	2.09730	61.743	15.64	4.433	15.80	4.401	3.605	1668	4732
3.72	963	2941	2553	3.583	628	8.232	2.09874	61.891	15.59	4.430	15.88	4.408	3.618	1647	4682
3.73	950	2876	2539	3.594	616	8.300	2.10017	62.038	15.55	4.426	15.96	4.415	3.632	1626	4632
3.74	937	2811	2525	3.604	604	8.374	2.10160	62.181	15.51	4.423	16.04	4.423	3.645	1605	4582
3.75	924	2746	2511	3.614	592	8.448	2.10303	62.324	15.47	4.420	16.12	4.430	3.659	1584	4532
3.76	911	2681	2497	3.625	580	8.522	2.10446	62.471	15.42	4.417	16.20	4.437	3.672	1563	4482
3.77	899	2616	2483	3.635	568	8.596	2.10589	62.618	15.38	4.414	16.28	4.444	3.686	1542	4432
3.78	886	2551	2469	3.645	556	8.670	2.10732	62.764	15.34	4.411	16.36	4.451	3.700	1521	4382
3.79	874	2486	2455	3.656	544	8.744	2.10875	62.910	15.30	4.408	16.4				

Force	Units	lbf	pdl	dyne	g	Newton
	1 lbf =	1	32.174	$4.44822 \times 10^{-5}$	453.59237	4.44822
	1 pdl =	0.0310809	1	$1.38255 \times 10^{-4}$	14.0981	0.138255
	1 dyne =	$2.24809 \times 10^{-6}$	$7.23301 \times 10^{-5}$	1	$1.01972 \times 10^{-3}$	$10^{-5}$
	1 g =	$2.20462 \times 10^{-3}$	0.070931	980.665	1	$9.80665 \times 10^{-3}$
	1 Newton =	0.224809	7.23298	$10^{-5}$	101.972	1

Length	Units	inch	feet	centimeter	meter
	1 in =	1	0.0833333	2.54	0.0254
	1 ft =	12	1	30.48	0.3048
	1 cm =	0.3937008	0.0328084	1	0.01
	1 m =	39.37008	3.28084	100	1

Density	Units	lbm/ft <sup>3</sup>	lbm/gal	g/cm <sup>3</sup>	kg/m <sup>3</sup>
	1 lbm/ft <sup>3</sup> =	1	0.11982646	0.01601847	16.01847
	1 lbm/gal =	7.480519	1	0.11982646	119.82646
	1 g/cm <sup>3</sup> =	62.42793	8.345402	1	$10^{-3}$
	1 kg/m <sup>3</sup> =	0.06242793	$8.345 \times 10^{-3}$	$10^{-3}$	1

Pressure	Units	lbf/in <sup>2</sup>	at 50 F	atm	kg/cm <sup>2</sup>	torr	bar	N/m <sup>2</sup>
	1 lbf/in <sup>2</sup> =	1	2.0897	0.06804596	0.0703066	51.71495	0.06894757	$6.894757 \times 10^{-3}$
	1 ft H <sub>2</sub> O, 60 F =	0.43309	1	0.0294703	0.0304495	22.3974	0.0290608	$2.98608 \times 10^{-3}$
	1 atm =	14.69595	33.9325	1	1.033227	760	1.01325	$1.01325 \times 10^{-5}$
	1 kg/cm <sup>2</sup> =	14.22335	32.8413	0.9678411	1	735.5596	0.980665	$9.80665 \times 10^{-4}$
	1 torr =	0.0193368	0.044648	$1.315789 \times 10^{-3}$	$1.359509 \times 10^{-3}$	1	$1.333224 \times 10^{-3}$	133.3224
	1 bar =	14.503775	33.4887	0.9869233	1.019716	750.0617	1	$10^{-5}$
	1 N/m <sup>2</sup> =	$1.450378 \times 10^{-4}$	$3.34887 \times 10^{-4}$	$9.869233 \times 10^{-6}$	$1.019716 \times 10^{-5}$	$7.500617 \times 10^{-3}$	$10^{-5}$	1

Figure A-5 Conversion Factors

Volume	Units	inch <sup>3</sup>	ft <sup>3</sup>	gal	liter	cm <sup>3</sup>	m <sup>3</sup>
	1 in <sup>3</sup> =	1	5.787037x10 <sup>-4</sup>	4.329004x10 <sup>-3</sup>	0.01638706	16.38706	1.638706x10 <sup>-5</sup>
	1 ft <sup>3</sup> =	1728	1	7.480519	28.31684	28316.84	0.02831684
	1 gal =	231	0.1336806	1	3.785411	3785.411	3.785411x10 <sup>-3</sup>
	1 liter =	61.02374	0.03531467	0.2641721	1	1000	10 <sup>-3</sup>
	1 cm <sup>3</sup> =	0.06102374	3.531467x10 <sup>-5</sup>	2.641721x10 <sup>-4</sup>	10 <sup>-3</sup>	1	10 <sup>-6</sup>
	1 m <sup>3</sup> =	61023.74	35.31467	264.1721	10 <sup>3</sup>	10 <sup>6</sup>	1

Mass	Units	lbm	gram (g)	kg	ton	metric ton
	1 lbm =	1	453.59237	0.045359237	0.0005	4.5359237x10 <sup>-4</sup>
	1 g =	2.204623x10 <sup>-3</sup>	1	10 <sup>-3</sup>	1.1023115x10 <sup>-6</sup>	10 <sup>-6</sup>
	1 kg =	2.204623	1000	1	1.1023115x10 <sup>-3</sup>	10 <sup>-3</sup>
	1 ton =	2000	907184.74	907.18474	1	0.9071846
	1 metric ton =	2204.623	10 <sup>6</sup>	1000	1.1023115	1

To Convert	Into	Multiply By
A		
Atmospheres	Ton/sq inch	0.007348
Atmospheres	cm of mercury	76
Atmospheres	dynes/cm <sup>2</sup>	1.013x10 <sup>6</sup>
Atmospheres	Ft of water (@ 4 degs C)	33.9
Atmospheres	In. of Mercury @ 0 degs C	29.92
Atmospheres	kgs/ sq cm	1.0333
Atmospheres	kgs/sq meter	10332
Atmospheres	lbf/sq ft (psf)	2116.4
Atmospheres	lbf/sq inch (psi)	14.7
Atmospheres	tons/ sq ft	1.058
B		
Bars	atmospheres	0.9869
Bars	dynes/ sq cm	10 <sup>6</sup>
Bars	kgs/sq meter	1.020x10 <sup>4</sup>
Bars	lbf/sq ft (psf)	2089
Bars	lbf/sq in (psi)	14.5
C		
Centigrade	fahrenheit	(C * 9/5) + 32
Centimeters (cms)	feet	3.281x10 <sup>-2</sup>
Centimeters	inches	0.3937
Centimeters	kilometers	10 <sup>-6</sup>
Centimeters	meters	0.01
Centimeters	miles	6.214x10 <sup>-6</sup>
Centimeters	millimeters	10
Centimeters	mils	393.7
Centimeters	yards	1.094x10 <sup>-2</sup>
Centimeters of Mercury	atmospheres	0.01316
Centimeters of Mercury	feet of water	0.4461
Centimeters of Mercury	kgs/sq meter	136
Centimeters of Mercury	lbf/sq ft (psf)	27.85
Centimeters of Mercury	lbf/sq inch (psi)	0.1934
Centimeters /sec	feet/min	1.9685
Centimeters /sec	feet/sec	0.03281
Centimeters /sec	kilometers/hr	0.036
Centimeters /sec	knots	0.1943

Figure A-5 Continued



To Convert	Into	Multiply By
Centimeters /sec	meter/min	0.6
Centimeters /sec	miles/hr	0.02237
Centimeters /sec	miles/min	$3.728 \times 10^{-4}$
Centimeters/sec/sec	feet/sec/sec	0.03281
Centimeters/sec/sec	kms/sec/sec	0.036
Centimeters/sec/sec	meters/sec/sec	0.01
Centimeters/sec/sec	miles/hr/sec	0.02237
Cubic centimeters	cubic feet (ft <sup>3</sup> )	$3.531 \times 10^{-5}$
Cubic centimeters	cubic inches (in <sup>3</sup> )	0.06102
Cubic centimeters	cubic meters (m <sup>3</sup> )	$10^{-6}$
Cubic centimeters	cubic yards (yd <sup>3</sup> )	$1.308 \times 10^{-6}$
Cubic centimeters	gallons (gal; US liq.)	$2.6242 \times 10^{-4}$
Cubic centimeters	liters	0.001
Cubic centimeters	pints (US liq.)	$2.113 \times 10^{-3}$
Cubic centimeters	quarts (US liq.)	$1.057 \times 10^{-3}$
Cubic feet	cubic centimeters	2832
Cubic feet	cubic inches	1.728
Cubic feet	cubic meters	0.02832
Cubic feet	cubic yards	0.03704
Cubic feet	gallon (US liq.)	7.48052
Cubic feet	liters	28.32
Cubic feet	pints (US liq.)	59.84
Cubic feet	quarts (US liq.)	29.92
Cubic inches	cubic centimeters	16.39
Cubic inches	cubic feet	$5.787 \times 10^{-4}$
Cubic inches	cubic meters	$1.639 \times 10^{-5}$
Cubic inches	cubic yards	$2.143 \times 10^{-5}$
Cubic inches	gallons	$4.329 \times 10^{-6}$
Cubic inches	liters	0.01639
Cubic inches	pints (US liq.)	0.03463
Cubic inches	quarts (US liq.)	0.01732
Cubic meters	cubic centimeter	$10^6$
Cubic meters	cubic feet	35.31
Cubic meters	cubic inches	61023
Cubic meters	cubic yards	1.308
Cubic meters	gallons (US liq.)	264.2
Cubic meters	liters	1000
Cubic meters	pints (US liq.)	2113
Cubic meters	quarts (US liq.)	1057

Figure A-5 Continued

To Convert	Into	Multiply By
Centimeters /sec	meter/min	0.6
Centimeters /sec	miles/hr	0.02237
Centimeters /sec	miles/min	$3.728 \times 10^{-4}$
Centimeters/sec/sec	feet/sec/sec	0.03281
Centimeters/sec/sec	kms/sec/sec	0.036
Centimeters/sec/sec	meters/sec/sec	0.01
Centimeters/sec/sec	miles/hr/sec	0.02237
Cubic centimeters	cubic feet (ft <sup>3</sup> )	$3.531 \times 10^{-5}$
Cubic centimeters	cubic inches (in <sup>3</sup> )	0.06102
Cubic centimeters	cubic meters (m <sup>3</sup> )	$10^{-6}$
Cubic centimeters	cubic yards (yd <sup>3</sup> )	$1.308 \times 10^{-6}$
Cubic centimeters	gallons (gal; US liq.)	$2.6242 \times 10^{-4}$
Cubic centimeters	liters	0.001
Cubic centimeters	pints (US liq.)	$2.113 \times 10^{-3}$
Cubic centimeters	quarts (US liq.)	$1.057 \times 10^{-3}$
Cubic feet	cubic centimeters	2832
Cubic feet	cubic inches	1.728
Cubic feet	cubic meters	0.02832
Cubic feet	cubic yards	0.03704
Cubic feet	gallon (US liq.)	7.48052
Cubic feet	liters	28.32
Cubic feet	pints (US liq.)	59.84
Cubic feet	quarts (US liq.)	29.92
Cubic inches	cubic centimeters	16.39
Cubic inches	cubic feet	$5.787 \times 10^{-4}$
Cubic inches	cubic meters	$1.639 \times 10^{-5}$
Cubic inches	cubic yards	$2.143 \times 10^{-5}$
Cubic inches	gallons	$4.329 \times 10^{-6}$
Cubic inches	liters	0.01639
Cubic inches	pints (US liq.)	0.03463
Cubic inches	quarts (US liq.)	0.01732
Cubic meters	cubic centimeter	$10^6$
Cubic meters	cubic feet	35.31
Cubic meters	cubic inches	61023
Cubic meters	cubic yards	1.308
Cubic meters	gallons (US liq.)	264.2
Cubic meters	liters	1000
Cubic meters	pints (US liq.)	2113
Cubic meters	quarts (US liq.)	1057

Figure A-5 Continued

To Convert	Into	Multiply By
Cubic yard	cubic centimeters	$7.646 \times 10^5$
Cubic yard	cubic feet	27
Cubic yard	cubic inches	46656
Cubic yard	cubic meters	0.7646
Cubic yard	gallons (US liq.)	202
Cubic yard	liters	764.6
Cubic yard	pints (US liq.)	1615.9
Cubic yard	quart (US liq.)	807.9

#### D

Days	seconds	86400
Degree (angle)	quadrants	0.01111
Degree (angle)	radian	0.01745
Degree (angle)	revolutions	$2.7778 \times 10^{-3}$
Degree (angle)	seconds	3600
Degree/sec	radian/sec	0.01745
Degree/sec	revolutions/min	0.1667
Degree/sec	revolutions/sec	$2.778 \times 10^{-3}$
Dyne sq cm	atmosphere	$9.869 \times 10^{-7}$
Dyne sq cm	inches of Mercury at 0 degs C	$2.953 \times 10^{-5}$
Dyne sq cm	inches of water at 4 degs C	$4.015 \times 10^{-4}$
Dynes	grams	$1.02 \times 10^{-3}$
Dynes	joules centimeter	$10^{-7}$
Dynes	joules meter (newton)	$10^{-5}$
Dynes	kilogram	$1.02 \times 10^{-6}$
Dynes	poundals	$7.233 \times 10^{-5}$
Dynes	pounds	$2.248 \times 10^{-6}$
Dynes	bar	$10^{-6}$

#### E

Ergs	Btu	$9.84 \times 10^{-11}$
Ergs	dyne-centimeters	1
Ergs	foot-pounds	$7.367 \times 10^{-8}$
Ergs	gram-calories	$0.2389 \times 10^{-7}$
Ergs	gram-centimeters	$1.020 \times 10^{-3}$
Ergs	horsepower-hrs	$3.725 \times 10^{-14}$
Ergs	joules	$10^{-7}$

Figure A-5 Continued

To Convert	Into	Multiply By
Ergs	kg-calories	$2.389 \times 10^{-11}$
Ergs	kg-meter	$1.02 \times 10^{-8}$
Ergs	kilowatt-hrs	$0.2778 \times 10^{-13}$
Ergs	watt-hrs	$0.2778 \times 10^{-10}$
Ergs/sec	Btu/min	$5.688 \times 10^{-9}$
Ergs/sec	ft-lbf/min	$4.427 \times 10^{-6}$
Ergs/sec	ft-lbf/sec	$7.3756 \times 10^{-8}$
Ergs/sec	horsepower	$1.341 \times 10^{-10}$
Ergs/sec	kg-calories/min	$1.433 \times 10^{-9}$
Ergs/sec	kilowatts	$10^{-10}$
F		
Feet	centimeters	30.48
Feet	kilometers	$3.048 \times 10^{-4}$
Feet	meters	0.3048
Feet	miles (nautical)	$1.645 \times 10^{-4}$
Feet	miles (statute)	$1.894 \times 10^{-4}$
Feet	millimeters	304.8
Feet	mils	$1.2 \times 10^{-4}$
Feet of water	atmosphere	0.0295
Feet of water	inches of Mercury	0.8826
Feet of water	kgs/sq centimeters	0.03048
Feet of water	kgs/ sq meter	304.8
Feet of water	lbf/sq feet (psf)	62.43
Feet of water	lbf/sq inch (psi)	0.4335
Feet/min	centimeters/sec	0.508
Feet/min	feet/sec	0.01667
Feet/min	kilometer/hr	0.01829
Feet/min	meters/min	0.3048
Feet/min	miles/hr	0.01136
Feet/sec	centimeters/sec	30.48
Feet/sec	kilometers/hr	1.097
Feet/sec	knots	0.5921
Feet/sec	meters/min	18.29
Feet/sec	miles/hr	0.6818
Feet/sec	miles/min	0.01136
Feet/sec/sec	cms/sec/sec	30.48
Feet/sec/sec	kms/hr/sec	1.097

Figure A-5 Continued

To Convert	Into	Multiply By
Feet/sec/sec	meters/sec/sec	0.3048
Feet/sec/sec	miles/hr/sec	0.6818
Foot-pounds	Btu	$1.286 \times 10^{-3}$
Foot-pounds	ergs	$1.356 \times 10^{-7}$
Foot-pounds	gram-calories	0.3238
Foot-pounds	hp-hrs	$5.05 \times 10^{-7}$
Foot-pounds	joules	1.356
Foot-pounds	kg-calories	$3.24 \times 10^{-4}$
Foot-pounds	kg-meters	0.1383
Foot-pounds	kilowatt-hrs	$3.766 \times 10^{-7}$
Foot-pounds	newton-meters	1.35582
Foot-pounds/min	foot-pound sec	0.01667
Foot-pounds/min	horsepower	$3.03 \times 10^{-5}$
Foot-pounds/min	kg-calories/min	$3.24 \times 10^{-4}$
Foot-pounds/min	kilowatts	$2.26 \times 10^{-5}$
Foot-pounds/sec	Btu/hr	4.6263
Foot-pounds/sec	Btu/min	0.07717
Foot-pounds/sec	horsepower	$1.818 \times 10^{-3}$
Foot-pounds/sec	kg-calories/min	0.01945
Foot-pounds/sec	kilowatts	$1.356 \times 10^{-3}$

G

Gallons	cubic centimeters	3785
Gallons	cubic feet	0.1337
Gallons	cubic inches	231
Gallons	cubic meters	$3.785 \times 10^{-3}$
Gallons	cubic yards	$4.951 \times 10^{-3}$
Gallons	liters	3.785
Gallons (liq. Br Imp)	gallons (US liq)	1.20095
Gallons (US)	gallons (imp)	0.83267
Gallons of water	pounds of water	8.3453
Grams	dynes	980.7
Grams	joules centimeter	$9.807 \times 10^{-5}$
Grams	joules meter (newtons)	$9.807 \times 10^{-3}$
Grams	kilogram	0.001
Grams	milligrams	1000
Grams	ounces ounce (troy)	0.03215
Grams	poundals	0.07093

Figure A-5 Continued

To Convert	Into	Multiply By
Grams	pounds	$2.205 \times 10^{-3}$
Grams/cm	pounds/inch	$5.6 \times 10^{-3}$
H		
Horsepower	Btu/min	42.44
Horsepower	foot-lbf/min	33000
Horsepower	foot-lbf/sec	550
Horsepower (metric)	hp (550 ft lbf/sec)	0.9863
Horsepower	hp metric (542.5 ft lbf/sec)	1.014
Horsepower	kg-calories/min	10.68
Horsepower	kilowatts	0.7457
Horsepower	watts	745.7
Horsepower (boiler)	Btu/hr	33.479
Horsepower (boiler)	kilowatts	9.803
I		
Inches	centimeters	2.54
Inches	meters	$2.54 \times 10^{-2}$
Inches	miles	$1.578 \times 10^{-5}$
Inches	millimeters	25.4
Inches	mils	1000
Inches	yards	$2.778 \times 10^{-2}$
Inches of Mercury	atmospheres	0.033422
Inches of Mercury	feet of water	1.133
Inches of Mercury	kgs/sq cm	0.03453
Inches of Mercury	kgs/sq meter	345.3
Inches of Mercury	pounds/sq ft (psf)	70.73
Inches of Mercury	pounds/sq in (psi)	0.4912
Inches of Water @ 4 degs C	atmospheres	$2.458 \times 10^{-3}$
Inches of Water @ 4 degs C	inches of Mercury	0.07355
Inches of Water @ 4 degs C	kgs/sq centimeter	$2.54 \times 10^{-3}$
Inches of Water @ 4 degs C	ounces/sq inches	0.5781
Inches of Water @ 4 degs C	lbf/sq ft (psf)	5.204
Inches of Water @ 4 degs C	lbf/sq in (psi)	0.03613

Figure A-5 Continued

To Convert	Into	Multiply By
J		
Joules	Btu	$9.840 \times 10^{-4}$
Joules	ergs	$10^7$
Joules	foot-pounds	0.7376
Joules	kg-calories	$2.389 \times 10^{-4}$
Joules	kg-meters	0.102
Joules	watts-hrs	$2.778 \times 10^{-4}$
Joules/cm	grams	$1.02 \times 10^4$
Joules/cm	dynes	$10^7$
Joules/cm	joules/meter (newtons)	100
Joules/cm	poundals	723.3
Joules/cm	pounds	22.48
K		
Kilograms (kgs)	dynes	980665
Kilograms	grams	1000
Kilograms	joules/cm	0.09807
Kilograms	joules/meter (newtons)	9.807
Kilograms	poundals	70.93
Kilograms	pounds	2.205
Kilograms	ton (long)	$9.842 \times 10^{-4}$
Kilograms	ton (short)	$1.102 \times 10^{-3}$
Kilogram/cubic meter	grams/cu cm	0.001
Kilogram/cubic meter	pounds/cu ft	0.06243
Kilogram/cubic meter	pounds/cu inch	$3.613 \times 10^{-5}$
Kilograms/meter	pounds/ft	0.672
Kilogram/sq cm	dynes	980665
Kilogram/sq cm	atmosphere	0.9678
Kilogram/sq cm	feet of water	32.81
Kilogram/sq cm	inches of Mercury	28.96
Kilogram/sq cm	lbf/sq feet (psf)	2048
Kilogram/sq cm	lbf/sq inch (psi)	14.22
Kilogram/sq meter	atmospheres	$9.678 \times 10^{-5}$
Kilogram/sq meter	bars	$98.07 \times 10^{-6}$
Kilogram/sq meter	feet of water	$3.281 \times 10^{-3}$
Kilogram/sq meter	inches of Mercury	$2.896 \times 10^{-3}$
Kilogram/sq meter	lbf/sq feet (psf)	0.2048
Kilogram/sq meter	lbf/sq inch (psi)	$1.422 \times 10^{-3}$
Kilometers (kms)	centimeters	$10^{-6}$

Figure A-5 Continued

To Convert	Into	Multiply By
Kilometers	feet	3281.0
Kilometers	inches	$3.937 \times 10^4$
Kilometers	meters	1000.0
Kilometers	miles	0.6214
Kilometers	millimeters	$10^6$
Kilometers	yards	1094.0
Kilometers/hr	centimeters/sec	27.78
Kilometers/hr	feet/min	54.68
Kilometers/hr	feet/sec	0.9113
Kilometers/hr	knots	0.5396
Kilometers/hr	meters/min	16.67
Kilometers/hr	miles/hr	0.6214
Kilometers/hr/sec	cms/sec/sec	27.78
Kilometers/hr/sec	ft/sec/sec	0.9113
Kilometers/hr/sec	meters/sec/sec	0.2778
Kilometers/hr/sec	miles/hr/sec	0.6214
Kilowatts	Btu/min	56.92
Kilowatts	horsepower	1.341
Knots	feet/hr	6080
Knots	kilometers/hr	1.08532
Knots	nautical miles/hr	1.0
Knots	statute miles/hr	1.151
Knots	yards/hr	2027.0
	feet/sec	1.689
L		
Liters	bushels (US dry)	0.02838
Liters	cubic cm	1000
Liters	cubic feet	0.03531
Liters	cubic inches	61.02
Liters	cubic meters	0.001
Liters	cubic yards	$1.308 \times 10^{-3}$
Liters	gallons (US liq)	0.2642
Liters	pints (US liq)	2.113
Liters	quarts (US liq)	1.057
Lumen	spherical candle power	0.07958
Lumen	watt	0.001496

Figure A-5 Continued



To Convert	Into	Multiply By
<b>M</b>		
Meters	centimeters	100
Meters	feet	3.281
Meters	inches	39.37
Meters	kilometers	0.001
Meters	miles (nautical)	$5.396 \times 10^{-4}$
Meters	miles (statute)	$6.214 \times 10^{-4}$
Meters	millimeters	1000
Meters	yards	1.094
Meters/min	cms/sec	1.667
Meters/min	feet/min	3.281
Meters/min	feet/sec	0.05468
Meters/min	kms/hr	0.06
Meters/min	knots	0.03238
Meters/min	miles/hr	0.03728
Meters/sec	feet/min	196.8
Meters/sec	feet/sec	3.281
Meters/sec	kilometers/hr	3.6
Meters/sec	kilometers/min	0.06
Meters/sec	miles/hr	2.237
Meters/sec	miles/min	0.03728
Meters/sec/sec	cms/sec/sec	100
Meters/sec/sec	ft/sec/sec	3.281
Meters/sec/sec	kms/hr/sec	3.6
Meters/sec/sec	miles/hr/sec	2.237
Microns	meters	$1.0 \times 10^{-6}$
Miles (nautical)	feet	6080.27
Miles (nautical)	kilometers	1.852
Miles (nautical)	meters	1853
Miles (nautical)	miles (statute)	1.1516
Miles (nautical)	yards	2027
Miles (statute)	centimeters	$1.609 \times 10^5$
Miles (statute)	feet	5280
Miles (statute)	inches	$6.336 \times 10^4$
Miles (statute)	kilometers	1.609
Miles (statute)	meters	1609
Miles (statute)	miles (nautical)	0.8684
Miles (statute)	yards	1760
Miles/hr	cms/sec	44.7
Miles/hr	feet/min	88

Figure A-5 Continued

To Convert	Into	Multiply By
Miles/hr	feet/sec	1.467
Miles/hr	kms/hr	1.609
Miles/hr	kms/min	0.02682
Miles/hr	knots	0.8684
Miles/hr	meters/min	26.82
Miles/hr	miles/min	0.1667
Miles/hr/sec	cms/sec/sec	44.7
Miles/hr/sec	feet/sec/sec	1.467
Miles/hr/sec	kms/hr/sec	1.609
Miles/hr/sec	meters/sec/sec	0.447
Miles/min	cms/sec	2682
Miles/min	feet/sec	88
Miles/min	kms/min	1.609
Miles/min	knots/min	0.8684
Miles/min	miles/hr	60
Millimeters	centimeters	0.1
Millimeters	feet	$3.281 \times 10^{-3}$
Millimeters	inches	0.03937
Millimeters	kilometers	$10^{-6}$
Millimeters	meters	0.001
Millimeters	miles	$6.214 \times 10^{-7}$
Millimeters	mils	39.37
Millimeters	yards	$1.094 \times 10^{-3}$
Mils	centimeters	$2.54 \times 10^{-3}$
Mils	feet	$8.333 \times 10^{-5}$
Mils	inches	0.001
Mils	kilometers	$2.54 \times 10^{-8}$
Mils	yards	$2.778 \times 10^{-5}$
Minutes (angles)	degrees	0.01667
Minutes (angles)	quadrants	$1.852 \times 10^{-4}$
Minutes (angles)	radians	$2.909 \times 10^{-4}$
Minutes (angles)	seconds	60
N		
Newtontons	Dynes	$1.0 \times 10^5$
Newtontons	pounds (lbf)	0.2248

Figure A-5 Continued

To Convert	Into	Multiply By
O		
Ounce	drams	16
Ounce	grains	437.5
Ounce	grams	28.349527
Ounce	pounds	0.0625
Ounce	ounces (troy)	0.9115
Ounce	tons (long)	$2.79 \times 10^{-5}$
Ounce	tons (metric)	$2.835 \times 10^{-5}$
Ounce (fluid)	cubic inches	1.805
Ounce (fluid)	liters	0.02957
Ounces (troy)	grains	480
Ounces (troy)	grams	31.103481
Ounces (troy)	ounce (avdp.)	1.09714
Ounces (troy)	pennyweights (troy)	20
Ounces (troy)	pounds (troy)	0.08333
Ounce/sq inch	Dynes/sq cm	4309
Ounce/sq inch	lbf/sq in (psi)	0.0625

P		
Parsec	miles	$19 \times 10^{12}$
Parsec	kilometers	$3.084 \times 10^{13}$
Pennyweight (troy)	grams	1.55517
Pennyweight (troy)	pounds (troy)	$4.1667 \times 10^{-3}$
Pints (dry)	cubic inches	33.6
Pints (liq)	cubic centimeters	473.2
Pints (liq)	cubic feet	0.01671
Pints (liq)	cubic inches	28.87
Pints (liq)	cubic meters	$4.732 \times 10^{-4}$
Pints (liq)	cubic yards	$6.189 \times 10^{-4}$
Pints (liq)	gallons	0.125
Pints (liq)	liters	0.4732
Pints (liq)	quarts (liq.)	0.5
Pounds (avoirdupois)	ounces (troy)	14.5833
Poundals	dynes	13826
Poundals	grams	14.1
Poundals	joules/cm	$1.383 \times 10^{-3}$
Poundals	joules/meter (newtons)	0.1383
Poundals	kilograms	0.0141
Poundals	pounds	0.03108

Figure A-5 Continued

To Convert	Into	Multiply By
Pounds	drams	256
Pounds	dynes	$44.4823 \times 10^{-4}$
Pounds	grains	7000
Pounds	grams	453.5924
Pounds	joules/cm	0.04448
Pounds	joules/meter (newtons)	4.448
Pounds	kilograms	0.4536
Pounds	ounces	16
Pounds	ounces (troy)	14.5833
Pounds	poundals	32.17
Pounds	pounds (troy)	1.21528
Pounds (lbf)	slugs	$3.1081 \times 10^{-2}$
Pounds	tons (short)	0.0005
Pounds (troy)	grains	5760
Pounds (troy)	grams	373.24177
Pounds (troy)	ounces (avdp.)	13.1657
Pounds (troy)	ounces (troy)	12
Pounds (troy)	pennyweights (troy)	240
Pounds (troy)	pounds (avdp)	0.822857
Pounds (troy)	tons (long)	$3.6735 \times 10^{-4}$
Pounds (troy)	tons (metric)	$3.7324 \times 10^{-4}$
Pounds (troy)	tons (short)	$4.1143 \times 10^{-4}$
Pounds of water	cubic feet	0.01602
Pounds of water	cubic inches	27.68
Pounds of water	gallons	0.1198
Pounds of water/min	cubic feet/sec	$2.670 \times 10^{-4}$
Pounds-feet	cm-dynes	$1.356 \times 10^{-7}$
Pounds-feet	cm-grams	13825
Pounds-feet	meter-kgs	0.1383
Pounds/cubic feet	grams/cubic cm	0.01602
Pounds/cubic feet	kgs/cubic meter	16.01847
Pounds/cubic feet	pounds/cubic inch	$5.787 \times 10^{-4}$
Pounds/cubic inch	grams/cubic centimeter	27.68
Pounds/cubic inch	kgs/cubic meter	$2.768 \times 10^{-4}$
Pounds/cubic inch	pounds/cubic feet	1728
Pounds/sq foot (psf)	atmospheres	$4.725 \times 10^{-4}$
Pounds/sq foot (psf)	bars	$4.788 \times 10^{-4}$
Pounds/sq foot (psf)	dyne/sq cm	$4.788 \times 10^{-2}$
Pounds/sq foot (psf)	feet of water (4 degs C)	0.01602

Figure A-5 Continued

To Convert	Into	Multiply By
Pounds/sq foot (psf)	inches of Mercury @ 0 degs C	0.01414
Pounds/sq foot (psf)	kgs/sq meter	4.882
Pounds/sq foot (psf)	lb/sq inch (psi)	$6.944 \times 10^{-3}$
Pounds/sq foot (psf)	newton sq. meter	47.88
Pounds/sq inch (psi)	atmospheres	0.06804
Pounds/sq inch (psi)	dynes/sq cm.	$6.8948 \times 10^4$
Pounds/sq inch (psi)	feet of water @ 4 degs C	2.307
Pounds/sq inch (psi)	inches of water @ 4 degs C	27.681
Pounds/sq inch (psi)	inches of Mercury @ 0 degs C	2.036
Pounds/sq inch (psi)	newton sq meter	$6.8948 \times 10^3$
Pounds/sq inch (psi)	kgs/sq meter	703.1
Pounds/sq inch (psi)	pounds/sq inch (psf)	144

#### Q

Quadrants (angle)	degree	90
Quadrants (angle)	minutes	5400
Quadrants (angle)	radians	1.571
Quadrants (angle)	seconds	$3.24 \times 10^5$
Quarts (dry)	cubic inches	67.2
Quarts (liq.)	cubic centimeters	946.4
Quarts (liq.)	cubic feet	0.03342
Quarts (liq.)	cubic inches	57.75
Quarts (liq.)	cubic meters	$9.464 \times 10^{-4}$
Quarts (liq.)	cubic yards	$1.238 \times 10^{-3}$
Quarts (liq.)	gallons	0.25
Quarts (liq.)	liters	0.9463

#### R

Radians	degrees	57.3
Radians	minutes	3438
Radians	quadrants	0.6366
Radians	seconds	$2.063 \times 10^5$
Radians/sec	degrees/sec	57.3
Radians/sec	revolutions/min	9.549
Radians/sec	revolutions/sec	0.1592
Radians/sec/sec	revs/min/min	573
Radians/sec/sec	revs/min/sec	9.549

Figure A-5 Continued

To Convert	Into	Multiply By
Radians/sec/sec	revs/sec/sec	0.1592
Revolutions	degrees	360
Revolutions	quadrants	4
Revolutions	radians	6.283
Revolutions/min	degree/sec	6
Revolutions/min	radians/sec	0.1047
Revolutions/min	revs/sec	0.01667
Revolutions/min/min	radians/sec/sec	$1.745 \times 10^{-3}$
Revolutions/min/min	revs/min/sec	0.01667
Revolutions/min/min	revs/sec/sec	$2.778 \times 10^{-4}$
S		
Seconds (angle)	degress	$2.778 \times 10^{-4}$
Seconds (angle)	minutes	0.01667
Seconds (angle)	quadrants	$3.087 \times 10^{-6}$
Seconds (angle)	radians	$4.848 \times 10^{-6}$
Slug	ibm	32.2
Slug	kgs	14.594
Square centimeters	circular mils	$1.973 \times 10^{-5}$
Square centimeters	square feet	$1.076 \times 10^{-2}$
Square centimeters	square inches	0.155
Square centimeters	square meters	0.0001
Square centimeters	square miles	$3.861 \times 10^{-11}$
Square centimeters	square millimeters	100
Square centimeters	square yards	$1.196 \times 10^{-4}$
Square feet	acres	$2.296 \times 10^{-5}$
Square feet	circular mils	$1.833 \times 10^{-8}$
Square feet	square centimeters	929
Square feet	square inches	144
Square feet	square meters	0.0929
Square feet	square miles	$3.587 \times 10^{-8}$
Square feet	square millimeters	$9.29 \times 10^{-4}$
Square feet	square yards	0.1111
Square inches	circular mils	$1.273 \times 10^{-6}$
Square inches	square centimeters	6.452
Square inches	square feet	$6.944 \times 10^{-3}$
Square inches	square millimeters	645.2
Square inches	square mils	$10^{-6}$

Figure A-5 Continued

To Convert	Into	Multiply By
Square inches	square yards	$7.716 \times 10^{-4}$
Square kilometers	acres	247.1
Square kilometers	square centimeters	$10^{10}$
Square kilometers	square feet	$10.76 \times 10^6$
Square kilometers	square inches	$1.55 \times 10^9$
Square kilometers	square meters	$10^4$
Square kilometers	square miles	0.3861
Square kilometers	square yards	$1.196 \times 10^6$
Square meters	acres	$2.471 \times 10^{-4}$
Square meters	square centimeters	$10^4$
Square meters	square feet	10.76
Square meters	square inches	1550
Square meters	square miles	$3.861 \times 10^{-7}$
Square meters	square millimeters	$10^6$
Square meters	square yards	1.196
Square miles	acres	640
Square miles	square feet	$27.88 \times 10^6$
Square miles	square kilometers	2.59
Square miles	square meters	$2.59 \times 10^6$
Square miles	square yards	$3.098 \times 10^6$
Square millimeters	circular mils	1973
Square millimeters	square centimeters	0.01
Square millimeters	square feet	$1.076 \times 10^{-5}$
Square millimeters	square inches	$1.55 \times 10^{-3}$
Square yards	acres	$2.066 \times 10^{-4}$
Square yards	square centimeters	8361
Square yards	square feet	9
Square yards	square inches	1296
Square yards	square meters	0.8361
Square yards	square miles	$3.288 \times 10^{-7}$
Square yards	square millimeters	$8.361 \times 10^5$

T

Tons (long)	kilograms	1016
Tons (long)	lbf	2240
Tons (long)	tons (short)	1.12
Tons (metric)	kilogram	1000
Tons (metric)	lbf	2205

Figure A-5 Continued

To Convert	Into	Multiply By
Tons (short)	kilogram	907.1848
Tons (short)	ounce	32000
Tons (short)	ounce (troy)	29166.66
Tons (short)	lbf	2000
Tons (short)	pounds (troy)	2430.56
Tons (short)	tons(long)	0.89287
Tons (short)	tons (metric)	0.9078
W		
Watts	Btu/hr	3.4129
Watts	Btu/min	0.05688
Watts	ergs/sec	107
Watts	foot-lbf/min	44.27
Watts	foot-lbf/sec	0.7378
Watts	horsepower	$1.341 \times 10^{-3}$
Watts	horsepower (metric)	$1.36 \times 10^{-3}$
Watts	kg-calories/min	0.01433
Watts	kilowatts	0.001
Y		
Yards	centimeters	91.44
Yards	kilometers	$9.144 \times 10^{-4}$
Yards	meters	0.9144
Yards	miles (nautical)	$4.934 \times 10^{-4}$
Yards	miles (statute)	$5.682 \times 10^{-4}$
Yards	millimeters	914.4

Figure A-5 Concluded



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## APPENDIX B

## APPENDIX B

### Tid Bits

This section offers tid bits of information that can be useful. The information listed below is not in any particular order.

- 1) One count of drag is 0.0001.
- 2) A  $\Delta$  drag count of 30 counts is a significant increase.
- 3) Wing sweep decreases  $C_{L_{\alpha}}$  and  $C_{L_{max}}$ .
- 4) Critical Mach is increased by wing sweep.
- 5) In a powered test (V/STOL), vary  $q$ , dynamic pressure, to obtain the appropriate thrust coefficient.
- 6)  $\alpha_{ZL}$  does not vary with aspect ratio.
- 7) Principal contributor to  $C_{l_{\beta}}$  (dihedral effect) is wing geometric dihedral, 'Z' position of wings and sweep angle.
- 8) Supercritical airfoils (wings) delay the drag rise due to shock formation.
- 9) When the Mach number is  $> 0.3$ , be sure to correct the dynamic pressure (density) for compressibility effects.

## Wind Tunnel First Aid Kit

This section is a collection of suggested items that would be helpful to the testing engineer if they accompanied him/her to the testing site. It keeps the testing engineer from losing an 'atta boy'.

- 1) Extra money for testing
- 2) Aspirin
- 3) Coffee mug
- 4) Triangle, ruler, symbol template, and other graphing equipment
- 6) Pencils with extra lead and erasers
- 7) Binders and dividers to put plots in
- 8) Labels & gummed tabs for binders and dividers
- 9) Paper clips
- 10) Calculator
- 11) Notebook for personal notes
  
- 12) Notebook for wind tunnel log. It is most important that the testing engineer write down everything that happens during the test. By the time the reduced data arrives back 'home', you will have forgotten what happened during the test.
  
- 13) Test Plan
- 14) Run Schedule
- 15) Stress report
- 16) Previous data for comparison
- 17) Model drawings
- 18) Reference material
- 19) Raucous reading material for the wee hours in the morning
- 20) Telephone numbers from back home and at the test site
- 21) Don't forget extra money for a post test party.

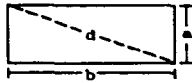
### Square



$$A = a^2$$

$$d = a\sqrt{2}$$

### Rectangle



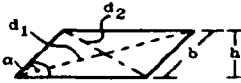
$$A = ab$$

$$d = \sqrt{a^2 + b^2}$$

$$I_x = \frac{ab^3}{12}$$

$$I_y = \frac{ba^3}{12}$$

### Parallelogram

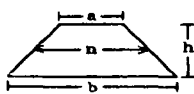


$$A = ab$$

$$d_1 = \sqrt{a^2 + b^2 + 2ab\cos\alpha}$$

$$d_2 = \sqrt{a^2 + b^2 - 2ab\cos\alpha}$$

### Trapezium



$$A = \left[ \frac{a+b}{2} \right] h = nh$$

$$n = \left[ \frac{a+b}{2} \right]$$

$$\bar{y} = \frac{h}{3} \left[ \frac{2a+b}{a+b} \right]$$

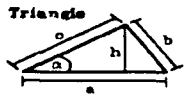
### Equilateral Triangle



$$A = \frac{a^2\sqrt{3}}{4}$$

$$h = \frac{a\sqrt{3}}{2}$$

### Triangle



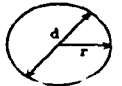
$$A = \left[ \frac{a+b}{2} \right] h = \frac{1}{2} absin\alpha$$

$$\bar{y} = \frac{h}{3}$$

$$\bar{x} = \frac{a+b}{3}$$

$$I_x = \frac{ah^3}{36}$$

### Circle



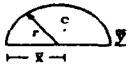
$$A = \pi r^2 = \frac{\pi}{4} d^2 = 0.785d^2$$

$$\text{Circum} = 2\pi r$$

$$I_x = \frac{\pi r^4}{4}$$

$$\bar{y} = \bar{x} = 0$$

### Semi-Circle

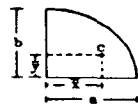


$$A = \frac{\pi r^2}{2}$$

$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_x = I_y = 0.1098r^4$$

### Semi-Parabolic

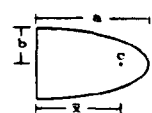


$$A = \frac{2}{3} ab$$

$$\bar{x} = \frac{2}{5} a$$

$$y = \frac{3}{8} b$$

### Parabolic

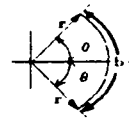


$$A = \frac{4}{3} ab$$

$$\bar{x} = \frac{2}{5} a$$

$$y = \frac{3}{8} b$$

### Circular Arc Sector



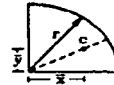
$$A = \frac{\theta r^2}{2}$$

$$b = 2\theta r$$

$$\bar{x} = \frac{2r\sin\theta}{3\theta}$$

$$\bar{y} = r\sin\theta$$

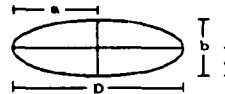
### Quarter Circular



$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_x = I_y = 0.0549r^4$$

### Ellipse

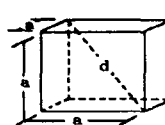


$$A = \frac{\pi}{4} Dd = \pi ab$$

$$\text{Circum} = \pi \left[ \frac{D+d}{2} \right]$$

$$I_x = \frac{\pi}{4} ad^3$$

### Cube

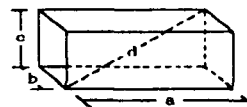


$$V = a^3$$

$$A = 6a^2$$

$$d = a\sqrt{3}$$

### Rectangle Parallelepiped



$$m = \text{mass}$$

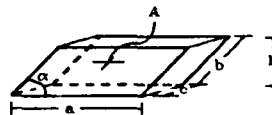
$$V = abc$$

$$A = 2(ab + ac + bc)$$

$$d = \sqrt{a^2 + b^2 + c^2}$$

$$I_{xx} = \frac{m}{12} (a^2 + b^2)$$

### Parallelepiped



$$V = hA_{\text{surf}} = abcsin\alpha$$

### Cylinder



$$V = \frac{\pi}{4} d^2 h$$

$$A_{\text{surf}} = 2\pi r(r + h)$$

$$I_{xx} = \frac{\pi r^4}{4} + \frac{m h^2}{12}$$

$$I_{yy} = \frac{\pi r^4}{4}$$

### Cone



$$V = \frac{\pi}{3} hr^2$$

$$A = \pi r \left[ \sqrt{r^2 + h^2} + r \right]$$

$$\bar{y} = \frac{h}{4}$$

$$\bar{x} = \frac{3}{4} h$$

### Sphere



$$V = \frac{4}{3} \pi r^3 = \frac{\pi}{6} d^3$$

$$A_{\text{surf}} = 4\pi r^2$$

Figure B-1 Geometric Equations

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## NOTES

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## APPENDIX C

## APPENDIX C

### Power On Symbols

#### Moment Arms

$l_1$  = Horizontal Ram Drag Arm

$l_2$  = Vertical Thrust Arm

$l_3$  = Vertical Ram Drag Arm

$l_4$  = Horizontal Thrust Arm

$l_5$  = Lateral Thrust Arm

#### Thrust Induced Aerodynamic Forces (Pwr ON - Pwr OFF)

$\Delta L$  Lift

$\Delta D$  Drag

$\Delta M$  Pitching moment

$\Delta RM$  Rolling moment

#### Total Coefficients

$C_L$  Lift coefficient,  $L/qS$

$C_D$  Drag coefficient,  $D/qS$

$C_M$  Pitching moment,  $M/qS\bar{c}$

$C_{RM}$  Rolling moment,  $RM/qS$

#### Incremental Thrust Coefficients

$C_T$  Thrust coefficient,  $T/qS$

$\Delta C_{L_T}$  Lift coefficient due to Thrust

$\Delta C_{D_T}$  Drag coefficient due to thrust

$\Delta C_{M_T}$  Pitching moment coefficient due to Thrust

### Symbols continued

$\Delta C_{RM_T}$  Rolling moment coefficient due to Thrust

$\Delta C_{YM_T}$  Yawing moment coefficient due to Thrust

$\Delta C_{M_D}$  Pitching moment due to Ram drag

$C_{D_R}$  Ram drag coefficient,  $\frac{M_i V}{qS}$

### Aerodynamic Coefficients (direct thrust effect removed)

$C_{L_{Aero}}$  or  $C_{L_A}$  Lift coefficient

$C_{D_{Aero}}$  or  $C_{D_A}$  Drag coefficient

$C_{M_{Aero}}$  or  $C_{M_A}$  Pitching moment coefficient

$C_{RM_{Aero}}$  or  $C_{RM_A}$  Rolling moment coefficient

### Miscellaneous

$x$  = Fan Number

$\theta_T$  = Total Thrust Vector (measured from longitudinal axis)

$\theta_x$  = Fan Thrust Vector

$\bar{c}$  = M.A.C.

$b$  = Wing Span

$h/D$  Non-dimensional height with respect  
to the nozzle diameter

Power On Aerodynamic Equations (ref. 5)

Incremental Aerodynamic Coefficient (x = fan number)

$$\theta_T = \sin^{-1} \Sigma T_x \sin \theta_x / \sqrt{(\Sigma T_x \cos \theta_x)^2 + (\Sigma T_x \sin \theta_x)^2}$$

$$\Delta C_{L_T} = C_T \sin(\theta_T + \alpha)$$

$$\Delta C_{D_T} = - C_T \cos(\theta_T + \alpha)$$

$$\Delta C_{M_{T_x}} = T_x (l_{4_x} \sin \theta_{T_x} + l_{2_x} \cos \theta_{T_x}) / q S \bar{c}$$

$$\Delta C_{M_T} = \Sigma \Delta C_{M_{T_x}} \quad (\text{Fan } 1, 2, 3, \dots, x)$$

$$\Delta C_{M_{D_x}} = D_R (l_{3_x} \cos \alpha + l_{1_x} \sin \alpha) / q S \bar{c}$$

$$\Delta C_{M_D} = \Sigma \Delta C_{M_{D_x}} \quad (\text{Fan } 1, 2, 3, \dots, x)$$

$$\Delta C_{RM_x} = T_x l_{5_x} [\sin \theta_x \cos \alpha + \cos \theta_x \sin \alpha] / q S \bar{c}$$

$$\Delta C_{YM_x} = T_x l_{5_x} [\cos \theta_x \cos \alpha - \sin \theta_x \sin \alpha] / q S \bar{c}$$

Aerodynamic Coefficients (total - thrust effects)

(figure III-5)

$$C_{L_{Aero}} = C_L - \Delta C_{L_T}$$

$$C_{D_{Aero}} = C_D - \Delta C_{D_T} - C_{D_R}$$

$$C_{M_{Aero}} = C_M - \Delta C_{D_T} - \Delta C_{M_D}$$

$$C_{RM_{Aero}} = C_{RM} - \Sigma \Delta C_{RM_x}$$

$$C_{YM_{Aero}} = C_{YM} - \Sigma \Delta C_{YM_x}$$

$$D_R = M_i V$$

$$C_{D_R} = \frac{D_R}{q S}$$

#### Induced Force Coefficients

$$\frac{\Delta L}{T} = \frac{C_{L A_{pwr\ on}} - C_{L_{pwr\ off}}}{C_T}$$

$$\frac{\Delta D}{T} = \frac{C_{D A_{pwr\ on}} - C_{D_{pwr\ off}}}{C_T}$$

$$\frac{\Delta M}{T \bar{c}} = \frac{C_{RM A_{pwr\ on}} - C_{RM_{pwr\ off}}}{C_T}$$

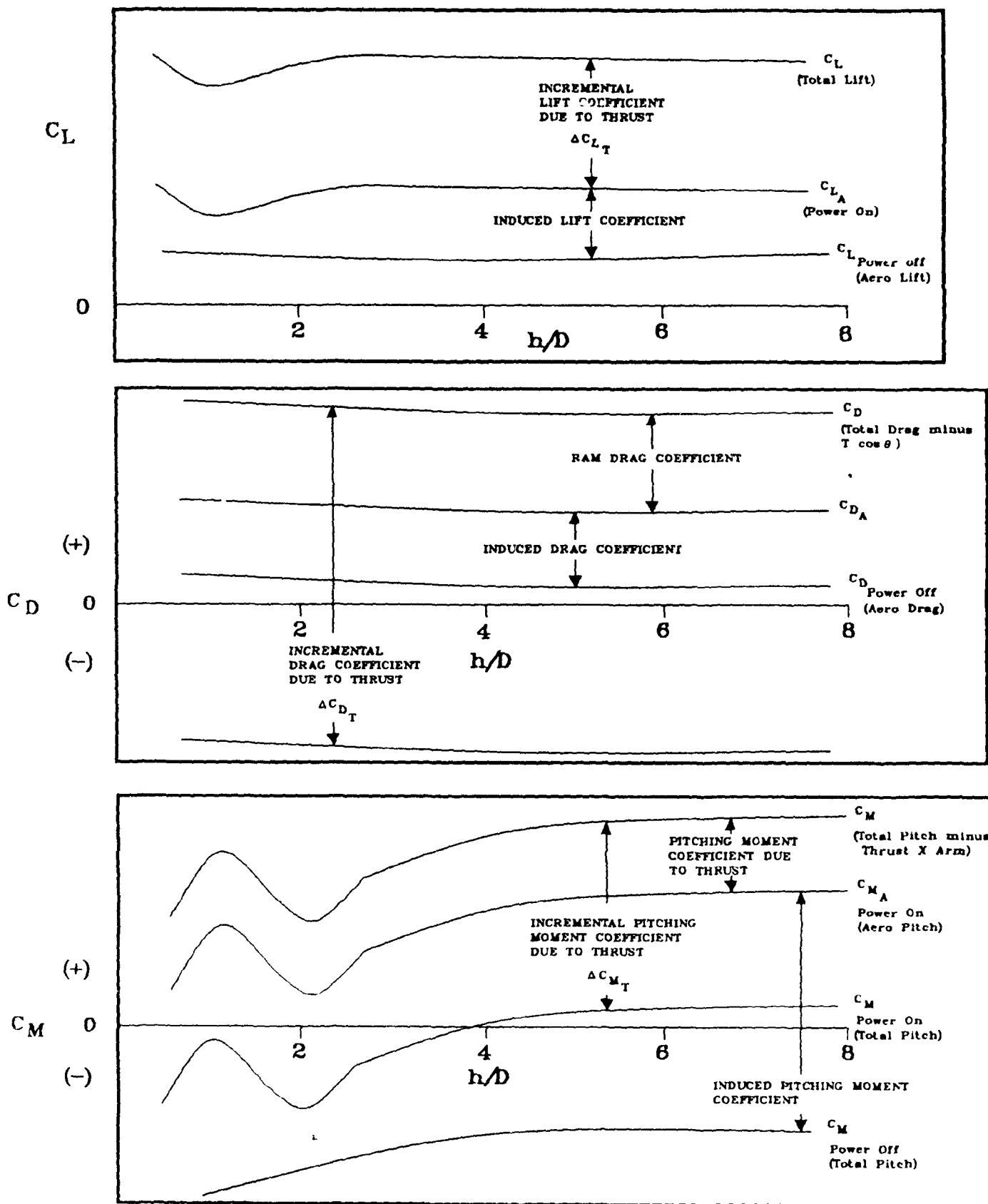


Figure C-1 Powered Effects

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## NOTES

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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**SUPPLEMENTARY**

**INFORMATION**



DEPARTMENT OF THE AIR FORCE

WRIGHT LABORATORY (AFMC)  
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FROM: WL/DOA  
Wright-Patterson AFB OH 45433-6523

7 January 1994

SUBJ: Notice of Changes in Technical Report(s) #WL-TR-91-3073/ADA240263

TO: Defense Technical Information Center  
Attn: DTIC-OCC  
Cameron Station  
Alexandria VA 22304-6145

Please change subject report as follows:

Include the attached 2 corrected pages in subject Tech Report  
as an ERRATA SHEET.

ERRATA ADA 240263

WM F. WHALEN  
Chief, STINFO and Technical Editing Branch  
Operations and Support Directorate

Cy to: WL/DOL (M. Kline)

Corrections to version 1.0 of Subsonic Wind Tunnel Testing Handbook  
 WL-TR-91-3073; DTIC AD A240263

- 1) Page II-2; Mean Aerodynamic Chord (MAC); Add a comma after 'span'.
- 2) Page IV-7; Figure IV-4, Top figure 'Aerodynamic Angles';  $-\beta$  is on the wrong plane. It should be on the  $+\psi$  plane.
- 3) Page IV-9; Balance to Body Axis , force equations (roll=0) [not matrix]; on  $C_{N_b}$  term ...  $-C_{Y_{bal}} \sin\beta \sin\alpha$  should be positive  $+C_{Y_{bal}} \sin\beta \sin\alpha$ .
- 4) Page IV-9; Balance to Body Axis , force equations (roll=0) [not matrix]; on  $C_{A_b}$  term ...  $-C_{Y_{bal}} \sin\beta \cos\alpha$  should be positive  $+C_{Y_{bal}} \sin\beta \cos\alpha$ .
- 5) Page IV-9; Balance to Body Axis , force equations (roll=0) [not matrix]; on  $C_{Y_b}$  term ...  $+C_{A_{bal}} \sin\beta$  should be negative  $-C_{A_{bal}} \sin\beta$ .
- 6) Page IV-13; Body Axis to Wind Axis, Force matrix; Change  $C_{S_b}$  to  $C_{Y_b}$ .
- 7) Page IV-14; Body Axis to Stability Axis, Force matrix; Change  $C_{S_b}$  to  $C_{Y_b}$ .
- 8) Page IV-15; Wind Axis to Stability Axis, Add a 'w' sub-subscript to  $C_L$  in the vector column of the force matrix.
- 9) Page V-2; change the units on 32.1741 to (lbm-ft/lbf-s<sup>2</sup>) from (ft/sec<sup>2</sup>).
- 10) Page VI-6; Figure VI-3;  $\eta_i$  is measured on this figure from the centerline to the inboard trailing edge flap chord.
- 11) Page VI-10; Inboard Section,  $b/2=26.96$ in; Change to  $b_i/2=26.96$  in.
- 12) Page VII-7, Method of Determining a Drag Polar; Change  $\sum_{i=1}^N C_L(i) * C_D(i)^2$  to  $\sum_{i=1}^N C_L(i)^2 * C_D(i)$ .
- 13) Page VII-7, Last line; change  $b=-2kC'$  to  $b=-2KC_L'$
- 14) Page VII-8; Resulting augmented matrix; Change first line from '3 1 3' to '5 1 0.3'; Change 0.11251 to 0.011251
- 15) Page XI-9; 4th line; Change 'in milli-volts [mv]' to 'in milli-volts [mv)]'

16) Page XI-12; Example Original Balance Calibration Matrix, no action is required. This is a point of clarification. Part of this matrix came from Figure XI-3. The original matrix was inadvertently entered and subsequently inverted in COLUMN order when in fact it should have been entered in ROW order. All the preceding calculations use the column order inverted matrix, but their ORDER of calculations and calculations is correct. If you "pretend" they were entered in row order everything works out fine. This was not corrected in this report.

17) Page A-3; Standard Atmosphere; The values used in this figure were obtained from a dated U.S. Standard Atmosphere source ie., speed of sound @ S.L. = 1107.64 when it should be 1116.1.

18) Delete Page A-12.

19) Page A-10...; Conversion Factors;

To Convert	Into	Multiply By	Change to
Centigrade	fahrenheit	$(C*9/5)-32$	$(C*9/5)+32$
Centimeters	kilometers	$10^{-6}$	$10^{-5}$
Cubic feet	cubic centimeters	2832	28,320.0
Cubic feet	Cubic inches	1.728	1728.0
Cubic inches	gallons	$4.329 \times 10^{-6}$	$4.329 \times 10^{-3}$
Degree/sec	revolutions/sec	$2.778 \times 10^3$	$2.778 \times 10^{-3}$
Ergs/sec	BTU/min	$5.688 \times 10^{-9}$	$5,688 \times 10^{-9}$
Hp (boiler)	BTU/hr	33.479	33,479.0
Knots	kilometers/hr	1.08532	1.8532
Knots	mautical mi/hr	1.0	nautical mi/hr

20) Page B-3; Figure B-1 Geometric Equations, Parallelogram; the length of 'a' is the base of the parallelogram.